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THEME

OPTIMAL DECENTRALIZED CONTROL DESIGN WITH OVERLAPPING STRUCTURE

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ABSTRACT

As many industrial systems have a high complexity; large scale systems became the subject of intensive research in systems and control theory. The complexity of the systems leads to severe difficulties that are encountered in the tasks of analyzing the system and designing and implementing appropriate control strategies algorithms. These difficulties arise mainly from dimensionality, uncertainty and information structure constraints. For these reasons the analysis and synthesis tasks cannot be solved economically in a single step as it is possible for similar analysis and design tasks small system. Therefore, it is common procedure in engineering practice to work with mathematical models that are simpler, but less accurate, then the best available model of a given physical process, "since the amount of computation required analyzing and controlling large scale system grows faster than its size. It has been long recognized that it is beneficial to decompose a large scale system into subsystems, and design controller for each subsystem independently on the basis of the local subsystems dynamics and the nature of their interconnections.

In this project, we have taken the advantages of overlapping decomposition technique and its application to design an optimal decentralized controller. We started by inclusion/contraction principle that allows us to create an equivalent higher order system then decomposition's theory to subdivide the whole expanded system into subsystems with lower dimensions and treat them independently. To show the efficiency of our algorithms we proposed optimal centralized control design and the decentralized one followed by a comparison.

Keywords: large scale system, overlapping decomposition, decentralized control, inclusion/ contraction, web winding system.

RESUME

Puisque les systèmes industriels sont complexes, les systèmes de grandes dimensions deviennent un sujet de recherche primordial dans le domaine de l'automatique. La complexité de ces systèmes pose des de sérieux problèmes d'algorithme de modélisation et de commande. Ces difficultés proviennent généralement de la dimensionnalité, de l'incertitude et de la structure des contraintes. Pour étudier ces systèmes à grandes dimension on se trouve souvent confronté à un ensemble important de données dont l'analyse devient impossible, la simulation trop couteuse et la réalisation d'une commande très difficile. Il est donc indispensable d'établir une théorie pour appréhender de manière systématique ces grands systèmes. Par conséquent, il est recommandé de décomposer ces grands systèmes et d'établir indépendamment la loi de commande recherchée pour chaque sous systèmes. Par conséquent, la résolution du problème de la commande décentralisée nécessite l'emploi d'outils adaptés et le choix de l'algorithme à mettre en œuvre dépend de la structure du système en considération.

Dans ce travail, l'approche proposée de la commande optimale décentralisée s'appuie sur la recherche du recouvrement des sous systèmes. L'exploitation de ces recouvrements s'étudient dans le cadre mathématique du principe de ' l'inclusion/contraction ' ce qui permet de définir un modèle augmenté via une transformation introduisent des redondances.
Pour mettre en œuvre notre approche, nous avons proposé deux algorithmes : l'un pour la commande optimale centralisée et l'autre pour la commande optimale décentralisée suivie d'une comparaison des performances.

Mots-clés: systèmes à grande dimension, décomposition avec recouvrement, commande décentralisée, inclusion/ contraction, système à trois moteurs.

ملخص

تتميز العديد من النظم الصناعية بالتعقيد و التي تأتي أساسا من وجود العديد من المتغيرات في مثل هذه الأنظمة، وأيضا من هيكل تداخل المعلومات القادمة إليها أو المنتقلة منها. من هذا يمكن أن تحصل مضاعفات تفقد جزء هام من المعلومات، بالإضافة إلى جعل دراسة هذه النظم - إذا لم يكن من المستحيل - على الأقل من الصعب جدا على الباحث.

للرياضيات العديد من التطبيقات في دراسة النظم الفيزيائية الصناعية ، سواء كانت بسيطة أو معقدة ، خطية أو غير خطية ،هذه التطبيقات ليست فقط في إنشاء نماذج للأنظمة ولكن تتجاوز ذلك إلى تبسيط المعادلات والحسابات المنشأة ، ثم تقليل الوقت محاكاة نماذج هاته النظم و التي تصنع الاختلاف في المجال الصناعي.

ناقش العديد من الباحثين تقنيات حديثة من أجل الاستفادة من هيكل النظام قيد الدراسة للتقليص من تلك التعقيدات مثل .D Siljack, Ikeda, Altug Iftar, Boccara, Nino واحدة من هذه التقنيات هو تداخل التحلل.

في هذا المشروع اتخذنا مزايا تقنية تداخل التحلل وتطبيقاتها كهدف أساسي لهذا البحث. لقد بدأنا بمبدأ إدراج / الانكماش الذي يسمح لنا بإنشاء نظام جديد ذو مرتبة أعلى ثم نظرية التحلل لتقسيم النظام برمته إلى نظم فرعية مع انخفاض الأبعاد والتعامل معها بشكل منفرد.

لتصميم وحدة تحكم، اخترنا نوعين: وحدة تحكم مركزية للنظام الأصلي المتداخل، وحدات تحكم مركزية للأنظمة الفرعية التي يتم جمعها لاحقا بالطريقة التي تضمن استقرار النظام برمته. هذان النوعان المختلفان من وحدات التحكم تسمح لنا بإجراء المقارنة واستخلاص مزايا كل أسلوب.

في النهاية، لتحسين النتائج التي حققت، أضفنا خوارزمية التحسين الأمثل للتحكم مركزي واللامركزي. وقد طبقت هذه الخوارزميات لنظامين مختلفين، الأول هو بناء مكون من ستة طوابق ذو درجتين من الحرية، والثاني هو نظام بساط صناعي متعرج ذو ثلاث محركات وأيضا درجتين من الحرية.

كلمات دالة : التحلل المتداخل ، مبدأ التحلل / الانكماش ، تحكم مركزي، تحكم لامركزي، نظام المبنى ، نظام البساط المتعرج.

Contents

Acronym and abbreviation

E.P	Equilibrium Point.
C.T.N.S	Continuous time nonlinear system.
MIMO	Multi-Input/ Multi-Output.
SISO	Single-Input/Single-Output.
O.P	Operating point.
L.S.S	Large scale system.
L.S.D.S	Large scale dynamic system.
C.C	Centralized Control.
D.C	Decentralized Control.
PID	Proportional Integral Derivative.
W.W.S	Web Winding System.
BBD	Bordered Block Diagonal.
P.I	Proportional Integral.
LTI	Linear time invariant.
LQR	Linear quadratic regulator.

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Introduction

Large scale systems are considered as very interesting area of study where most of real industrial physical systems are found very complicated; these complications come first from the mathematical model then analysis of this model, thus, it complicates study of such type of systems.

Many strategies have been developed for dealing with difficulties such as overlapping decomposition which is based on the idea of subdividing the whole system into smaller subsystems less difficult in study than the original one. However, this technique have some condition to apply, the challenge in such studies is to find real system comply with these conditions.

Large scale systems are really difficult to be controlled globally without the simultaneous development of numerically efficient algorithms for convex optimization. Our objective will be to develop an optimization algorithm that allows us to decompose the complex systems into lower interconnected subsystems that can be stabilized locally then conclude about global stability of the whole system.

The algorithm developed in this research is composed of different steps

First, we have applied inclusion/ contraction principle and we have checked if its conditions are satisfied, then we have verified the type of interconnection (weakly coupled), decomposition of system can be done and decentralized controller is designed.

To make this work has sense; we made comparison between centralized control and decentralized control designed for the same system.

Report of this project is composed of four chapters

In the first chapter, the control structures of large scale systems is briefly reviewed, in order to facilitate the understanding of this work, also decomposition technique approach explained with particular focus on the overlapping systems. At the end of this chapter, we finish up with application of overlapping algorithm in decomposing large scale system in order to design decentralized controller.

In the second chapter, overlapping decomposition technique is presented, where we start by inclusion and contraction principle and then we propose conditions that guarantee contractibility of controllers. After that we pass to optimal control with different performance index. At the end, we have defined a method combine the overlapping decomposition technique with decentralized control structure for quadratic performance index of optimal large scale system; we concluded that this method could be advantageous in such type of systems.

In chapter three, we present examples for which decentralized optimal overlapping decomposition technique given in the previous chapter is realizable. The first example is six floor building system with two degree of freedom. The second is web winding system with three motors and also two degree of freedom. The same algorithm has been applied for the two examples.

In chapter four, we present the MATLAB’s graphical user interface which is used to interface the algorithm developed in the whole work, then we show the detailed steps for the user to use correctly these two simulated systems and find out successfully the results according to the need of the user. At the end we discuss the results found for both building and web winding systems.

Finally, this work ends up with a comparison between optimal centralized and optimal decentralized control results in which the goal, steps of the work and the results are summarized then suggestions are made for future projects.

Chapter one

Control Structure and Decomposition of Complex Systems

In this chapter, the control structures of large scale systems is briefly reviewed in order to facilitate the understanding of this work, also decomposition technique approach is explained with particular focus on the overlapping systems.

At the end of this chapter, we finish up with application of overlapping algorithm in decomposing large scale system in order to design decentralized controller.

I.1. Introduction

In this chapter we deal with decomposition based on mathematical model for a widely known class of complex systems. A large scale system is one of the most important classes of real systems where the subsystems share a certain number of state variables. Such type of systems can be studied with two different strategies; the first in which we have to deal with the whole system as case study which is very complicated according to the structure of the system, the second strategy is based on aggregation-decomposition where the complex system can be expanded to larger one, then decomposed it into simple subsystems that can be easily treated.

I.2. Large scale systems (L.S.S)

A large-scale dynamical system is a system that is characterized by a large number of variables representing it, a strong interaction between subsystem variables, and a complex interaction between subsystems ([1],[2]), the stabilization problem of large-scale system has been one of the most popular research topics in control systems during the last decades (see [3,4,5,11,12]).

The structure of the controller that stabilizes large scale system is generally depending on the structure of controlled system itself. Most of large scale systems are nonlinear; these nonlinearities come from the interaction between states of each subsystem. In order to apply the decomposition techniques; it is necessary to linearise it around equilibrium points (E.P).

I.3. Linearisation of L.S.S

A continuous –time nonlinear system (C.N.S) is generally described by differential equation of the form

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \end{cases} \tag{1.1}$$

where

$t \in [t_0, \infty]$.

$x = x(t)$: is the state of the system belonging to (generally bounded) region $\Omega_x \subset R^n$.

u(t) : is the control inputs vector belonging to another (usually bounded) region $\Omega_u \subset R^m$ (often $m \le n$).

f, g : are nonlinear differentiable functions.

$y = y(t) \in R^l$: are the outputs of the system with $1 \le l \le n$.

When $n, l \succ 1$ the system is multi-input/multi-output (MIMO) system, while if $n = l = 1$, it is called a single input/single output (SISO) system, MISO and SIMO are similarly defined [13].

In order to analyze a model described by a nonlinear differential equation of the form (1.1), we first have to determine its E.Ps and study the behavior of the system near these points. Under certain conditions, this behavior is qualitatively the same as the behavior of a linear system. We therefore begin this section with a brief study of linear differential equations [14].

The point $Ep(x_{eq}, u_{eq})$ where $x_{eq} \in R^n, u_{eq} \in R^m$, is said to be an E.P of system (1.1) if it satisfies the following equation:

$$f(x_{eq}, u_{eq}) = 0 \tag{1.2}$$

Without loss of generality, it will be assumed everywhere in this work that $(x_{eq}, u_{eq}) = (0,0)$ is an E.P of System (1.1), with $f(0) = 0$ and $g(0) = 0$ such that, at the equilibrium, the output is zero; this assumption can be easily verified by change of state written in (1.2) [15].

A linear time-invariant approximation to this nonlinear system is valid in a region around the operating point (0.P) at $t = t_0, x(t_0) = x_0$, and $u(t_0) = u_0$. If the values of the system's states, $x(t)$ and inputs $u(t)$ are close enough to the operating point, the system will behave approximately linearly.

So to describe the linearised model, it helps to first define a new set of variables centered about the O.P of the states, inputs, and outputs:

$$\begin{cases} \delta x(t) = x(t) - x_{eq} \\ \delta u(t) = u(t) - u_{eq} \\ \delta y(t) = y(t) - y_{eq} \end{cases} \tag{1.3}$$

where

$$\begin{cases} \delta x(t) = x(t) - x_{eq}\big|_{t=t_0} = 0 \\ \delta u(t) = u(t) - u_{eq}\big|_{t=t_0} = 0 \\ \delta y(t) = y(t) - y_{eq}\big|_{t=t_0} = 0 \end{cases} \tag{1.4}$$

The linearized state space equations (around the equilibrium point 0) written in terms of $\delta x(t), \delta u(t)$ and $\delta y(t)$ are:

$$\begin{cases} \delta \dot{x}(t) = A\delta x(t) + B\delta u(t) \\ \delta y(t) = C\delta x(t) + D\delta u(t) \end{cases} \tag{1.5}$$

where A, B, C, and D are constant coefficient matrices. These matrices are defined as the Jacobians of the system, evaluated at the 0.P

$$A = \frac{\partial f}{\partial x}\bigg|_{t_0, x_0, u_0} \quad B = \frac{\partial f}{\partial u}\bigg|_{t_0, x_0, u_0} \quad C = \frac{\partial g}{\partial x}\bigg|_{t_0, x_0, u_0} \quad \text{and } D = \frac{\partial g}{\partial u}\bigg|_{t_0, x_0, u_0} \tag{1.6}$$

with

$$\begin{cases} x_0 = 0 \\ u_0 = 0 \end{cases} \tag{1.7}$$

By putting $\delta x(t) = x_n(t)$, $\delta u(t) = u_n(t)$ and $\delta y(t) = y_n(t)$; equa (1.5) can be written as

$$\begin{cases} \dot{x}_n(t) = Ax_n(t) + Bu_n(t) \\ y_n(t) = Cx_n(t) + Du_n(t) \end{cases} \tag{1.8}$$

I.4. Expansion-Contraction principle

Dynamic systems can be decomposed into overlapping subsystems which have common parts [7]; these L.S.Ss can be expanded into larger state space equations where the subsystems are stacked on the top of each other; so a new higher dimensional system is created.
The expanded system contains the essential information about motions of the original system; these informations can be extracted from the expansions by applying conventional technique devised for standard disjoint decomposition [7].

I.4.1. Expansion

Consider the linearized system defined by (1.8) (with $D=[0]$), where $x_n \in \Re^n, u_n \in \Re^m$ and $y_n \in \Re^l$ are respectively state, input and output vectors.

and consider the system described by

$$\begin{cases} \dot{x}_e = A_e x_e + B_e u_e \\ y_e = C_e x_e \end{cases} \quad (1.9)$$

where
$x_e \in \Re^{n_e}, u_e \in \Re^{m_e}$ and $y_e \in \Re^{l_e}$ are respectively, state, input and output vectors of system (1.9).

Assumptions should be taken:

- ✓ We assume that $n_e \geq n, m_e \geq m$ and $l_e \geq l$.
- ✓ Also it is assumed that y_n and y_e : are measurable.

The system (1.9) is an expansion of the system (1.8) (or (1.8) is contraction of (1.9)) if there exist full-rank transformations

$$\begin{cases} T : \Re^n \to \Re^{n_e} \\ N : \Re^m \to \Re^{m_e} \\ S : \Re^l \to \Re^{l_e} \end{cases} \quad (1.10)$$

such that for any initial state $x_0 \in \Re^n$ of the system (1.8) and any input $u_e(t) \in \Re^{m_e}$ of the system (1.9) we have

$$\begin{cases} x_{e0} = Tx_0 \\ u(t) = Nu_e(t) \end{cases} \Rightarrow \begin{cases} x_e(t; x_{e0}; u_e) = Tx(t; x_0; u) \\ y_e(t, x_e) = Sy(t; x) \end{cases}, \forall t \geq 0 \quad (1.11)$$

If the system (1.9) includes the system (1.8), then (1.9) is said to be an extension of (1.8), and (1.8) is called a contraction of (1.9) [8].

I.4.2. Necessary and sufficient condition of expansion

The necessary and sufficient conditions for expansion are provided by the following theorems

Theorem 1 [16]

The system (1.9) is an expansion of the system (1.8) if and only if there exist full-rank matrices $T \in \Re^{n_e} \times \Re^{n}, N \in \Re^{m} \times \Re^{m_e}, S \in \Re^{l_e} \times \Re^{l}$ such that

$$\begin{cases} TA = A_e T \\ TBN = B_e \\ SC = C_e T \end{cases} \tag{1.12}$$

The matrices of the original and expanded systems are related by

$$\begin{cases} A_e = TAT^I + E_A \\ B_e = TBN + E_B \\ C_e = SCT^I + E_c \end{cases} \tag{1.13}$$

where T^I is the pseudo inverse of T satisfying $T^I T = I_n$

$$T^I = (T^T T)^{-1} T^T \tag{1.14}$$

$E_A \in \Re^{n_e \times n_e}, E_B \in \Re^{n_e \times m_e}$ and $E_C \in \Re^{l_e \times n_e}$ are constant complementary matrices.

In order for the system (1.9) to be an expansion of (1.8); the complementary matrices must satisfy certain conditions provided by the following theorem.

Theorem 2 [16]

Let the matrices of (1.8) and (1.9) be related as (1.13); then the system (1.9) is an expansion of the system (1.8) if and only if:

$$\begin{cases} E_A T = 0 \\ E_B = 0 \\ E_c T = 0 \end{cases} \tag{1.15}$$

I.4.3. Contraction

A dual concept of expansion is contraction; we say that system (1.8) is contraction of system (1.9) if there exist a contraction matrix T with full row rank such that for any initial state x_0 of (1.6):

$$x_{e0} = T^{I} x_0 \Rightarrow x(t, x_0) = T x_e(t, x_{e0}), \forall t \geq 0 \tag{1.16}$$

I.5. Control structure

Large scale systems can be found in many industrial systems because its variables (ex pressure, temperature, and quality of production) should be controlled simultaneously; which means there exist always interactions between them.
There are two types of variables for any industrial multivariable system

- Manipulation variables.
- Controlled variables.

Branching scheme of these variables can be done in two different ways:

I.5.1. Centralized control

In this approach the control algorithm use all measured variables to generate the manipulation variables. Centralized control (C.C) is the traditional control method in which we consider all the states and interconnections as a single dynamic system. For large-scale dynamic system (L.S.D.S), the controller in a multivariable control framework is high order, which makes it difficult to implement in real time. Feedback control is used to reject disturbance and to improve performance. Many control strategies, such as PID [17, 18], loop shaping [19], gain-scheduling [20], multivariable H_∞ control [21] are applied in C.C design in industrial practice.

I.5.2. Decentralized control

In this configuration scheme, each manipulation variable branch to one controlled variable. It is first utilized in the industrial domain where many strategies of this type were successfully manipulated.

When decentralized control (D.C) is applied to large scale control systems, the interactions among control stations and the modeling are the major problems for the controller design. In the decentralized case, the interconnections between segments are usually neglected for control design purposes. The decentralized approach then greatly reduces the computation of hardware requirements. The goal of the method is to design a controller, which minimizes the influence of the remaining system and to guarantee the desired dynamics and stability of the total system.

The advantage of this method is that there are no any measurements of the quantities of coupling. It is necessary to know where the quantities of coupling are active in the subsystem. However, the designed control is robust against changes of the parameters in a limited range. Figure.1.1 is a 3-tension-zone web winding system, (W.W.S) in which each tension zone is designed by a decentralized separate controller [21].

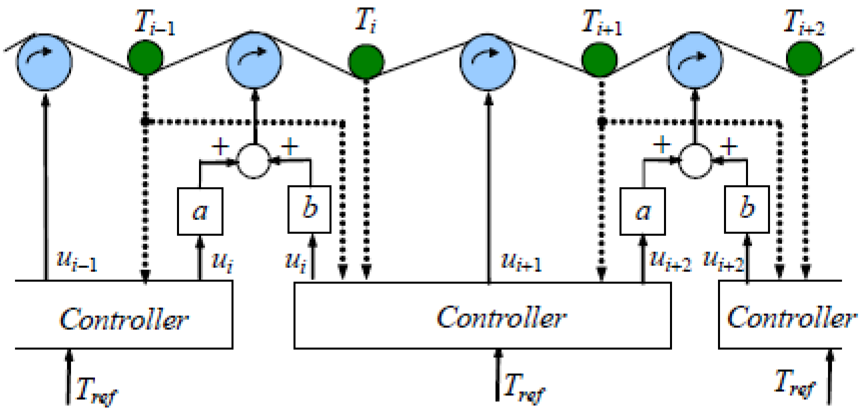


Fig.1.1. Decentralized control approach of web winding system (W.W.S)

There are two (02) aspects related to this type of configuration

- The simple loop of control is an independent algorithm; there is no communication between controllers.
- There exist always interactions that should be analyzed between loops (Appendix. B).

I.6. Decomposition of complex system

Systems that involve large number of variables are difficult to consider as one abject of process. Despite the high efficiency of modern computers, the formidable complexity of a large system can make the problem numerically intractable even with the most valuable one shot techniques. It has long been recognized that certain complex systems made of interacting elements can be decomposed into subsystems of lower dimensionality.

Decomposition of structural models has been one of the basic issues in the theory of large complex systems. Besides dimensionality and uncertainty, information structure constraint has been considered as one of the fundamental characteristics of system complexity [9]. The separate solutions of the subsystems are combined together in some way to provide a solution system while the decomposition principle can bring about a great saving in solution time over solving the whole system in one piece, it is still highly dependent on the choice of decomposition' type.

I.7. Decomposition' types

I.7.1. Decomposition using dynamic graphs

In this method, graphs are used in order to differentiate between the subsystems without modifying anything about the global system or its mathematical structure.

A "dynamic graph" is one of the most important methods that allow us decompose a complex system:

❖ The dynamic graphs

Let us consider the couple $G(V,E)$ with

V : is a set of vertices V_i

E : represents a set of edges e_j .

$$\begin{cases} V = (v_1, v_2, ..., v_m) \\ E = (e_1, e_2, ..., e_l) \end{cases} \tag{1.17}$$

G : is nonlinear function relates V and E , for $m = 3, l = 2$; equa (1.17) is written as

$$\begin{cases} V = (v_1, v_2, v_3) \\ E = (e_1, e_2) \end{cases}$$

and can be represented by

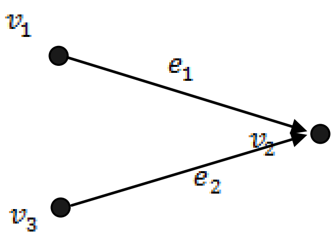


Fig.1.2. Example of dynamic graph

❖ Associated matrix of the directed graphs

It is a Boolean matrix $A(a_{ij})$ that can represent the directed graph, where:

$$a_{ij} = \begin{cases} 1 & \text{if there exist an oriented arc} \\ 0 & \text{elsewhere} \end{cases} \tag{1.18}$$

❖ Strong connectivity

We say that two components v_i and v_j are strongly connected if there exists –at least- one direct path from v_i returning to v_i and passing through v_j , and $\{v_i, v_j\}$ are considered as one strongly connected component.

❖ Algorithm of dynamic graph decomposition

- Find all the strongly connected components $s_1, s_2, ..., s_m$ of the directed graphs.
- Condensate the directed graphs (D) according to the strongly connected components.
- Use the topological order algorithm to reorder D.

- After that the associated matrix of the model is written in diagonal form, and we can say that the modal of the system is decomposed.

I.7.2. Bordered Block Diagonal (BBD) decomposition

Any sparse system can be reordered in the Bordered Block Diagonal form (at least, in principle), identifying an appropriate permutation matrix turns out to be a difficult graph-theoretic problem.

Consider the nonlinear system described by

$$\dot{x} = f(x) + Bu \tag{1.19}$$

where $f \in \Re^n \times \Re^n$ is a differentiable function that satisfies $f(0) = 0$.
As explained in section (I.3), this system can be linearized around equilibrium point (x_0) to find

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1.20}$$

where

$$A = Jacobian(f(x))\big|_{x=x_0} \tag{1.21}$$

In the following we will present an algorithm that simultaneously permutes matrices *A*, *B* and *C* in a way that induces a suitable BBD structure for the gain matrix. An important feature of this algorithm is that the resulting diagonal blocks have approximately equal sizes (such decomposition is said to be balanced) [6] and can be applied for strongly coupled systems as well as weakly coupled one.

The decomposition algorithm consists of three steps, the first of which is designed to provide an initial partitioning where the border is larger than any of the diagonal blocks. The border is then recursively decreased in Step 2, until only two large blocks remain. This procedure is repeated within each new block, ultimately producing a multilevel nested BBD structure. In the final stage of the decomposition, the local borders from the different levels are aggregated into a single overall border. This ensures that all diagonal blocks in the matrix will have approximately equal dimensions.
Structures of this type are highly desirable from a computational standpoint, since they minimize the communication overhead, and allow for an equal distribution of the workload among different processors and/or controllers.

We took the assumptions that the matrix $A(a_{ij})$ is sparse and symmetric, which means $a_{ij} \neq 0$ if and only if $a_{ji} \neq 0$

Step 1

The first Step in the decomposition requires the construction of an *initial border set* B_O, which is defined as

$$B_O = \{x_i \in X : \deg(x_i) \geq d_m\} \qquad (1.22)$$

where

d_m : is a preselected threshold value.
$N_{\max}$: is chosen freely, maximal allowable block size.

The only restriction is that $N_{\max} \leq |B_O|$ (this constraint secures that the border is larger than any of the diagonal blocks).

Step 2

After Step 1 is executed, the original graph is decomposed into a border set B_O^* and a number of disjoint blocks whose size does not exceed $(N_{\max})$. The objective of Step 2 is to recursively decrease set B_O^* for as long as at least two diagonal blocks remain. The procedure is then repeated within each individual block, thus creating a multilevel nested BBD structure.

Step 3

The procedure executed in Step 2 is controlled by a decomposition graph which contains all the necessary information about the nested border and block sizes.

I.7.3. Decomposition using gain matrix

- **Epsilon (ε)decomposition**

In the context of dynamic systems, epsilon decompositions are closely associated with the notions of expansion and contraction [7]. The algorithm for identifying epsilon decompositions can be applied to nonlinear dynamic systems of the form

$$\dot{x} = f(x) + Bu \qquad (1.23)$$

where $f \in \Re^n \times \Re^n$ is a differentiable function that satisfies $f(0) = 0$.

The basic idea in this case is to decompose the Jacobians $f(x)$ (evaluated at the origin), consider the permutation matrix $V(n_e \times n)$ where $(n_e \geq n)$ such that

$$Vf^{'}(0) = A_e V \tag{1.24}$$

We can always construct a mapping $f_e : \Re^{n_e} \times \Re^{n_e}$ such that:

$$f_e(V_x) = Vf(x) \quad , \ \forall x \in \Re^n$$

and

$$f_e^{'}(0) = A_e$$

If we define now: $h_e(x_e) = f_e(x_e) - A_e x_e$ **,** $B_e = VB$

So, the system will be

$$\dot{x}_e = A_{eD} x_e + \varepsilon A_{eC} + h_e(x_e) + B_e u \tag{1.25}$$

where

$$x_e(t, t_0, Vx_0) = Vx(t, t_0, x_0)$$

for an $x_0 \in \Re^n$.

When this condition is satisfied, we say that system (1.25) includes (1.19). It is important to recognize that this decomposition is highly desirable when dealing with L.S.Ss, since the block-diagonal structure of A_{eD} can be readily exploited for the design robust Decentralized controllers.

- **Input–Output Constrained Decompositions**

It is desirable that each diagonal block of the gain matrix A have at least one input and one output associated with it. When this is the case, the subsystems defined by bloc $A_{ii}\,(i = 1, 2, ..., N)$ are said to be input/output decomposition.

In the case of epsilon decomposition, this property can generally be ensured by reducing the value of parameter **ε,** which leads to larger blocks in matrix A_D (and therefore increases the likelihood that the corresponding blocks of B and C will be non-empty). It is important to recognize, however, that BBD decompositions do not allow for this kind of flexibility, since **ε** is set to zero by default. In order to resolve such problems, it is necessary to simultaneously permute matrix A into the BBD form and secure a compatible nonzero structure for matrices B and C .

The algorithm described in this section shows how such an objective can be accomplished in a systematic manner. We will assume that $V = X \cup U \cup Y$ consists of three disjoint sets of vertices

The state set $X = \{x_1, x_2, ..., x_N\}$

The input set $U = \{u_1, u_2, ..., u_l\}$

The output set $Y = \{y_1, y_2, ..., y_q\}$

then

$$(x_i, x_j) \in E \quad \text{if and only if} \quad a_{ij} \neq 0$$
$$(x_i, u_j) \in E \quad \text{if and only if} \quad b_{ij} \neq 0$$
$$(x_i, y_j) \in E \quad \text{if and only if} \quad c_{ij} \neq 0$$

As in the previous section, we can assume without loss of generality that matrix A is structurally symmetric; if that were not the case, we could simply use $A+A^T$ to obtain the decomposition. The proposed algorithm can now be described by the following sequence of steps

Step 1

The decomposition is initialized by identifying the shortest path from every input vertex to an output vertex. If any outputs remain unvisited at this point, the procedure is reversed with these outputs as the starting vertices. The initial graph is then formed as a union of the obtained paths.

Step 2

The second step begins by forming incidence sets

$$X_u^{(i)} = \{x_k \in X : (x_k, u_i) \in E\} \tag{1.26}$$

and

$$X_y^{(i)} = \{x_k \in X : (x_k, y_i) \in E\} \tag{1.27}$$

for each $u_i \in U$ and $y_i \in Y$ the union of these sets:

$$\bar{X} = \left\{\bigcup_{i=1}^{m} X_u^{(i)}\right\} \cup \left\{\bigcup_{i=1}^{q} X_y^{(i)}\right\} \tag{1.28}$$

is then added to the graph obtained in Step 1.

If there are any elements $x_i, x_j \in \bar{X}$ such that $(x_i, x_j) \in E$; these edges are added to the graph as well. Once the new graph Γ is formed, its disjoint components

$$\Gamma_i = X_i \cup Y \quad (i = 1, 2, ..., k) \tag{1.29}$$

are identified (note that by construction each set Γ_i contains at least one input and one output vertex).

It is important to recognize at this point that all vertices $x_i \in X$ that are incident to some input or output have been included in sets $\Gamma_i\,(i=1,2,...,k)$. Vertices $x_i \in \Gamma$ constitute set X_c, which represents a separator for the graph (this implies that the removal of X_c from the overall graph results in disjoint sets Γ_i).

Step3

In most problems, the separator X_C is large and needs to be reduced in order to obtain a meaningful BBD decomposition. To perform the necessary reduction, we first need to identify a subset of inputs and outputs that we wish to associate with the border block. For simplicity, let those inputs be contained in sets Γ_{k-1} and Γ_k we then, form the initial border B_O as the union of sets X_c, Γ_{k-1}, and Γ_k, The remaining sets $\Gamma_i\,(i=1,2,...,k-2)$ obviously represent the corresponding initial diagonal block, and the vertices of X_c are now reconnected to these blocks one by one. The order in which the reconnection proceeds is determined by the same greedy strategy that was described in the previous section. As in the case of balanced BBD decompositions, the reconnection continues for as long as at least two diagonal blocks remain.

- Reconnecting any further vertices from the border would require merging blocks 1 and 2. In that sense, the obtained border is minimal.

- The decomposition secures that each of the diagonal blocks has at least one input and one output associated with it, since only vertices belonging to X_C are reconnected. In addition, the preselected inputs and outputs remain associated with the border block.

- The reconnection scheme uses the same data structures and tiebreaking criteria as the one described in the previous section.

- The fact that each diagonal block has at least one input associated with it does not automatically guarantee controllability, or even input reachability. Along these lines, we should note that it is possible to construct a graph that its input is reachable, but generically uncontrollable due to dilation.

I.8. Summary

In this chapter, we first start by definition of large scale systems. Then we gave a whole description of expansion – contraction technique applied for linearised complex systems, and then we passed to most famous control structure applied for such kind of systems, after that we showed a brief review of different decomposition’s types such as epsilon decomposition, BBD decomposition and overlapping decomposition.

Finally, we concluded that it is possible to mix decomposition technique for appropriate control structure to solve the complexity problem of large scale systems.

Chapter two

Overlapping Decomposition and Optimization Problem of Complex Systems

In this chapter, overlapping decomposition technique is presented, where we have started by inclusion and contraction principle and then we propose conditions that guarantee contractibility of controllers. After that we pass to optimal control with different performance index.

At the end, we have defined a method that combine the overlapping decomposition technique with decentralized control structure for quadratic performance index of optimal large scale system; we have concluded that this method could be advantageous in such type of systems.

II.1. Introduction

Most of industrial systems can be described by large-scale models. There is a great deal of interest in performance analysis and control synthesis for L.S.Ss. A key practical consideration in designing a controller for this type of system is to rely on local information as much as possible. D.C theory was introduced in the previous chapter to address this consideration, and reduce the complexity of the control implementation for L.S.Ss. A decentralized controller consists of a number of isolated local controllers corresponding to the subsystems of the L.S.S. local control of lower dimensional subsystems increases system robustness with respect to a wide variety of structured and unstructured perturbations.

Decomposition is a prerequisite for D.C, breaking down a large system into a number of subsystems of lower dimensions. There are decomposition methods for L.S.Ss convenient for parallel computations, such as nested epsilon decompositions, balanced BBD decompositions (see section I.7)

When the system represents a structure of distinct interconnected elements (subsystems) which have physical meaning such as electric power networks, then in addition to the numerical simplification, the decomposition provide information about the important structural properties of the system. On the other hand, a reason for decomposition can be entirely numerical, in this case various transformations before and after decomposition can remove all physical meanings of the variables involved , and only the final solution of the overall system is to be of the variables involved , and only the final solution of the overall system is to be interpreted.

II.2. Overlapping decomposition

To formally introduce the notion of overlapping, let us consider a linear (linearized) system

$$S: \quad \dot{x} = Ax + Bu \tag{2.1}$$

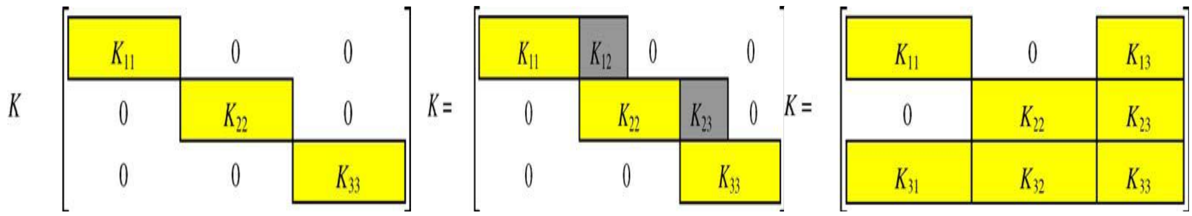
where $x(t) \in R^n$ is the state, $u(t) \in R^m$ represents the input, and matrices A and B have the structure:

$$A = \left[\begin{array}{cc:c} A_{11} & A_{12} & A_{13} \\ \hdashline A_{21} & A_{22} & A_{23} \\ \hdashline A_{31} & A_{32} & A_{33} \end{array}\right], B = \left[\begin{array}{c:c} B_{11} & B_{12} \\ \hdashline B_{21} & B_{22} \\ \hdashline B_{31} & B_{32} \end{array}\right] \tag{2.2}$$

where x: is composed of three components x_1, x_2 and x_3 with dimensions n_1, n_2 and n_3, so that $x = (x_1^T, x_2^T, x_3^T)^T$ and $n = n_1 + n_2 + n_3$ [25].

where the sub matrices A_{ij} have dimension $(n_i \times n_j)$, $i, j = 1, 2, 3$.

The structural constraints on the gain matrix of a L.S.S always follow the structure of the system itself.



1 2 3

Fig.2.1. Structural constraints on the gain matrix A, **1:** block-diagonal gain matrix structure. **2:** An overlapping gain matrix structure. **3:** BBD gain matrix structure.

Now, let us introduce the transformation matrices $\begin{cases} T:\Re^{n} \to \Re^{n_e} \\ N:\Re^{m} \to \Re^{m_e} \end{cases}$ such that

$$\begin{cases} TA = A_e T \\ TBN = B_e \end{cases} \tag{2.3}$$

In order to apply decomposition, it is necessary to derive the overlapping system (2.2) into non-overlapping system written as

$$A_e = \left[\begin{array}{cc:cc} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ \hdashline A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{array}\right],\quad B_e = \left[\begin{array}{c:c} B_{11} & B_{12} \\ B_{21} & B_{22} \\ \hdashline B_{21} & B_{22} \\ B_{31} & B_{32} \end{array}\right] \tag{2.4}$$

to do this, we chose complementary matrices as

$$E_A = \frac{1}{2}\begin{bmatrix} 0 & A_{12} & -A_{12} & 0 \\ 0 & A_{22} & -A_{22} & 0 \\ 0 & -A_{22} & A_{22} & 0 \\ 0 & -A_{32} & A_{32} & 0 \end{bmatrix} \tag{2.5}$$

so (2.3) can be written as

$$\begin{cases} A_e = TAT^{I} + E_A \\ B_e = TBN + E_B \end{cases} \tag{2.6}$$

Now, we arrange the three components of the state x into two overlapping components $x_{e1} = (x_1^T, x_2^T)^T$ and $x_{e2} = (x_2^T, x_3^T)^T$, which we use to form a new vector $x_e = (x_{e1}^T, x_{e2}^T)^T$ and induce an overlapping decomposition of A as indicated by dashed lines in (2.2). By the linear transformation, the vector x_e can be written as:

$$x_e = Tx \tag{2.7}$$

where T is the $n_e \times n$ matrix defined by:

$$T = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \tag{2.8}$$

and $n_e = n_1 + 2n_2 + n_3$ and I_1, I_2, I_3 are the identity matrices with dimensions corresponding to the components x_1, x_2 and x_3 of x [26].
The matrix $T^I = (T^T T)^{-1} T^T$ is chosen as the pseudo-inverse of T

$$T^I = \begin{bmatrix} I_1 & 0 & 0 & 0 \\ 0 & .5I_2 & .5I_2 & 0 \\ 0 & 0 & 0 & I_3 \end{bmatrix} \tag{2.9}$$

and $T^I T = I_n$.
The expanded system S_e can now be connected into two interconnected subsystems S_{e1} and S_{e2} defined by:

$$\begin{cases} S_{e1} : \dot{x}_{e1} = A_{e11} x_{e1} + A_{e12} x_{e2} + B_{e11} u_{e1} + B_{e12} u_{e2} \\ S_{e2} : \dot{x}_{e1} = A_{e21} x_{e1} + A_{e22} x_{e2} + B_{e21} u_{e1} + B_{e22} u_{e2} \end{cases} \tag{2.10}$$

where

$$A_{e11} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad A_{e12} = \begin{bmatrix} 0 & A_{13} \\ 0 & A_{23} \end{bmatrix}, \quad A_{e21} = \begin{bmatrix} A_{21} & 0 \\ A_{31} & 0 \end{bmatrix}, \quad A_{e22} = \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix}$$

with A_{e12}, A_{e21} are interconnection matrices as it is shown in Fig.2.2 [27].

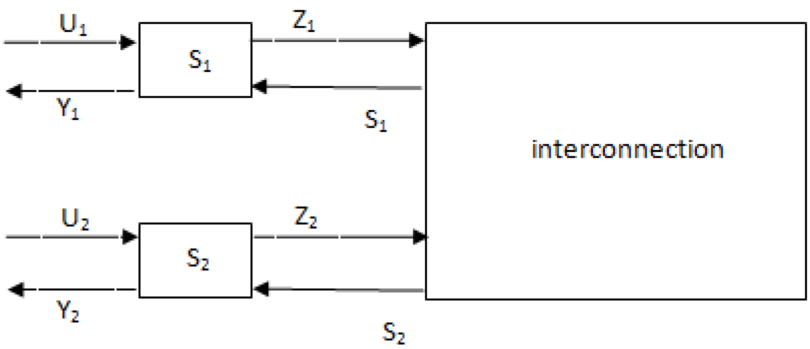


Fig.2.2. Representation of system (2.10) with its interactive blocks.

By comparing (2.10) with (2.4) we conclude that the disjoint decomposition of the expanded matrix A_e corresponds to the overlapping decomposition of the original matrix A as indicated by the dashed lines.
The class of expanded system S_e, which corresponds to the system S, can be obtained by choosing appropriately the transformation matrix T and the complementary matrix E_A.

E_A, E_B Should be chosen such that $\begin{cases} E_A T = 0 \\ E_B = 0 \end{cases}$.

If we choose the matrices T and E_A to be:

$$T=\begin{bmatrix} I_1 & 0 & 0 \\ 0 & .5I_2 & 0 \\ 0 & .5I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \quad T^I=\begin{bmatrix} I_1 & 0 & 0 & 0 \\ 0 & I_2 & I_2 & 0 \\ 0 & 0 & 0 & I_3 \end{bmatrix} \quad E_A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ .5A_{21} & .5A_{22} & -.5A_{22} & -.5A_{23} \\ -.5A_{21} & -.5A_{22} & .5A_{22} & .5A_{23} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.11}$$

We get the expanded system S_e with the matrix A_e defined as

$$A_e=\left[\begin{array}{cc|cc} A_{11} & A_{12} & A_{21} & A_{13} \\ A_{21} & A_{22} & 0 & 0 \\ \hline 0 & 0 & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{32} & A_{33} \end{array}\right] \tag{2.12}$$

The two distinct expanded matrices (2.4) and (2.12), which are obtained from the same matrix (2.2), are characterized by the identity of their diagonal blocks. This means that the decomposition of the expanded systems results into identical decoupled subsystems [7].

II.3. Optimal control

Optimization is a very desirable feature in day-to-day life. We like to work and use our time in an optimum manner, use resources optimally and so on. The subject of optimization is quite general in the sense that it can be viewed in different ways depending on the approach (algebraic or geometric), the interest (single or multiple), the nature of the signals (deterministic or stochastic), and the stage (single or multiple) used in optimization. As we notice that the calculus of variations is one small area of the big picture of the optimization field, and it forms the basis for our study of optimal control systems. Further, optimization can be classified as static optimization and dynamic optimization.

- **Static Optimization**

It is concerned with controlling a plant under steady state conditions, i.e., the system's variables are not changing with respect to time. The plant is then described by algebraic equations. Techniques used are ordinary calculus, Lagrange multipliers, linear and nonlinear programming.

- **Dynamic Optimization**

It concerns with the optimal control of plants under dynamic conditions, i.e., the system variables are changing with respect to time and thus the time is involved in system description. Then the plant is described by differential equations.

II.3.1. Formulation of the optimal control problem

The "optimal control problem" is to find a control $u^* \in U$ which causes the system

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \end{cases} \quad (2.13)$$

to follow a trajectory $x^* \in X$ that minimizes the performance measure

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g_O(x(t), u(t), t)dt \quad (2.14)$$

where

$x(t)$: is a $n \times 1$ state vector.

$u(t)$: is $p \times 1$ input vector.

f : is a vector- valued function.

II.3.2. Performance index (P.I) of familiar optimization problems

- **P.I for Time-Optimal Control System**

We try to transfer a system from an arbitrary initial state $x(t_0)$ to a specified final state $x(t_f)$ in minimum time. The corresponding P.I is:

$$J = \int_{t_0}^{t_f} dt = t_f - t_0 \quad (2.15)$$

- **P.I for Fuel-Optimal Control System**

Consider a spacecraft problem, Let $u(t)$ be the thrust of a rocket engine and assume that the magnitude $|u(t)|$ of the thrust is proportional to the rate of fuel consumption. In order to minimize the total expenditure of fuel, we may formulate the P.I as

$$J = \int_{t_0}^{t_f} |u(t)|\, dt \quad (2.16)$$

For several controls, we may write it as

$$J = \int_{t_0}^{t_f} \sum_{i=1}^{m} R_i \, |u_i(t)| dt \quad (2.17)$$

where R: is a weighting factor.

- **P.I for Minimum-Energy Control System**

Consider $u_i(t)$ as the current in the i^{th} loop of an electric network. Then $\sum_{i=1}^{m} u_i^2(t) r_i dt$ (where r_i is the resistance of the i^{th} loop) is the total power or the total rate of energy expenditure of the network. Then, for minimization of the total expended energy, we have a performance criterion as

$$J = \int_{t_0}^{t_f} \sum_{i=1}^{m} u_i^2(t) r_i dt \tag{2.18}$$

or in general

$$J = \int_{t_0}^{t_f} u(t)^T Ru(t) dt \tag{2.19}$$

where R : is a positive definite matrix.

Similarly, we can think of minimization of the integral of the squared error of a tracking system. We then have;

$$J = \int_{t_0}^{t_f} x^T(t) Q x(t) dt \tag{2.20}$$

where $x_d(t)$ is the desired value, $x_a(t)$ is the actual value, and $x(t) = x_a(t) - x_d(t)$, is the error. Here, Q is a weighting matrix, which can be positive semi-definite.

- **P.I for Terminal Control System**

In a terminal target problem, we are interested in minimizing the error between the desired target position $x_d(t_f)$ and the actual target position $x_a(t_f)$ at the end of the maneuver or at the final time t_f . The terminal (final) error is $x(t_f) = x_a(t_f) - x_d(t_f)$.
Taking care of positive and negative values of error and weighting factors, we structure the cost function as

$$J = x^T(t_f) F x(t_f) \tag{2.21}$$

which is also called the terminal cost function. Here, F is a positive semi-definite matrix [28].

- **P.I for General Optimal Control System**

Combining the above formulations, we have a P.I in general form as

$$J = \frac{1}{2}x^T(t_f)Fx(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left[x^T(t)Qx(t) + u^T(t)Ru(t)\right]dt \qquad (2.22)$$

or

$$J = S(x(t_f), t_f) + \int_{t_0}^{t_f} V(x(t), u(t), t)dt \qquad (2.23)$$

To give the state $x^*(t)$ that optimizes the general P.I defined by (2.23) with some constraints on the control variables $u(t)$ and/or the state variables $x(t)$. The final time t_f may be fixed, or free, and the final (target) state may be fully or partially fixed or free. The entire problem statement is also shown pictorially in Figure 2.3.

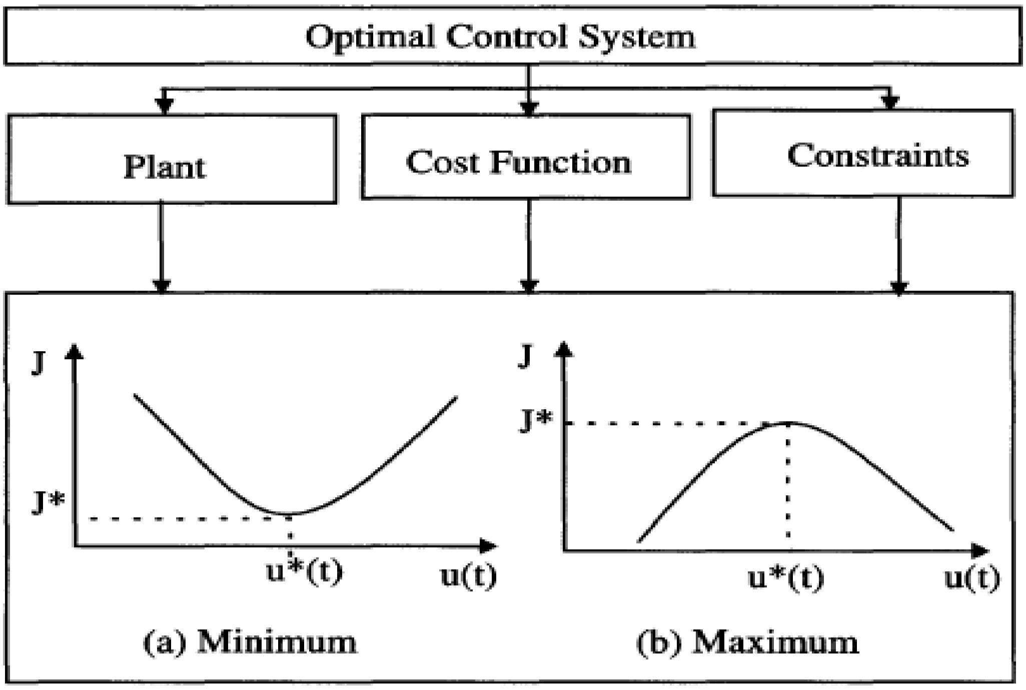


Fig.2.3. Optimal control problems

III.4. Inclusion-contraction of optimal systems

Extension-contraction technique (explained previously) can be applied for performance indices of optimal control systems.
Consider the system S defined by (2.1), with which we associate the P.I:

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \qquad (2.24)$$

where

Q : is positive semi-definite matrix.
R : is positive definite matrix.

Consider also the system S_e defined in (2.10), with which we associate the P.I

$$J_e = \int_0^\infty (x_e^T Q x_e + u_e^T R u_e) dt \tag{2.25}$$

The pair (S_e, J_e) includes the pair (S, J) if there exist transformation $\begin{cases} T : \Re^n \to \Re^{n_e} \\ N : \Re^{m_e} \to \Re^m \end{cases}$ such that S_e includes S

i.e:

$$\begin{cases} x(t, x_0, Nu_e) = T^I x_e(t, Tx_0, u_e) \\ J(x_0, Nu_e) = J_e(Tx_0, u_e) \end{cases} \quad \text{for all } t \geq 0 \tag{2.26}$$

for any initial state $x_0 \in \Re^n$ and for any input $u_e \in \Re^{m_e}$ [29].

II.5. Contractibility of controller

We can design decentralized optimal controllers for the original system by formulation of the concept of the inclusion principle presented in the previous chapter and extend it to the original system. This requires a contraction of the controllers that are calculated in the extended spaces, they are to be implemented in the original spaces. Some necessary and sufficient conditions have to be satisfied to contract these controllers. In this chapter we are going to present them. Dynamic controllers are considered as dynamic systems whereas static controllers are just gains that are fed back from the output.

II.5.1. Problem statement

Let us consider the LTI controller for the system S described by

$$\Psi : \begin{cases} \dot{z} = Fz + Gy \\ v = Hz + Ky \end{cases} \tag{2.27}$$

where $z \in \Re^p$ is the state and $v \in \Re^m$ is the output of the controller Ψ; the output v of Ψ is applied to the input of the system S in order to control it; i.e:

$$u = v$$

Also, consider the LTI controllers:

$$\Psi_{ei} : \begin{cases} \dot{z}_{ei} = F_{ei} z_{ei} + G_{ei} y_{ei} \\ v_{ei} = H_{ei} z_i + K_{ei} y_e \end{cases} \quad i = 1, 2, ..., n \tag{2.28}$$

For the decentralized system described by (2.28) $z_{ei} \in \Re^{p_{ei}}$ are the states and $v_{ei} \in \Re^{m_{ei}}$ is the output of the i^{th} decentralized controller Ψ_{ei} and m_{ei} is the dimension of the i^{th} input n_{ei} of the system S_e. The output v_{ei} is applied to the i^{th} input of the system S_e for control purposes.

$$u_{ei} = v_{ei} \quad , \ i = 1, 2, ..., n \tag{2.29}$$

The overall expanded controller, which consists of decentralized controllers $\Psi_{ei}, i = 1, 2, ..., n$, can be compactly described as

$$\Psi_e : \begin{cases} \dot{z}_e = F_e z_e + G_e y_e \\ v_e = H_e z_e + K_e y_e \end{cases} \tag{2.30}$$

where the dimension p of Ψ is less than or equal the dimension p_e of Ψ_e; this is reasonable because S is a part of S_e and should not require larger dimension controller.

$$\begin{cases} z_e = \left[z_{e1}^T, z_{e2}^T, ..., z_{en}^T \right] \\ v_e = \left[v_{e1}^T, v_{e2}^T, ..., v_{en}^T \right] \\ F_e = blockdiag(F_{e1}, F_{e2}, ..., F_{en}) \\ G_e = blockdiag(G_{e1}, G_{e2}, ..., G_{en}) \\ H_e = blockdiag(H_{e1}, H_{e2}, ..., H_{en}) \\ K_e = blockdiag(K_{e1}, K_{e2}, ..., K_{en}) \end{cases} \tag{2.31}$$

The controller (2.28) is contractible to the controller (2.27), if there exist full rank transformation

$$\begin{cases} T : \Re^n \rightarrow \Re^{n_e} \\ N : \Re^{m_e} \rightarrow \Re^m \\ \mathrm{M} : \Re^{p_e} \rightarrow \Re^p \end{cases}$$

such that for any initial state $x_0 \in \Re^n$ of the system S, for any input $u_e(t) \in \Re^{m_e}, 0 \le t \le \infty$ of the system S_e, and for any initial state $z_{eo} \in \Re^{p_e}$ of the controller Ψ_e, It satisfies:

$$\begin{cases} x_{e0} = Tx_0 \\ u(t) = Nu_e(t) \quad \forall t \ge 0 \\ z_0 = Mz_{e0} \end{cases} \Rightarrow \begin{cases} z(t; z_0, y, u) = Mz_e(t; z_{e0}, y_e, u_e) \\ u(t; z, y) = Nv_e(t; z_e, y_e) \end{cases} \quad \forall t \ge 0 \tag{2.32}$$

Note that, since the controller is designed in the expanded spaces $u_e(t)$ must be arbitrary (otherwise, there will be restrictions on the controllers that could be designed) [8].

II.5.2. Necessary and sufficient conditions for contractibility of controllers

- **Theorem (1)**

Suppose that S_e is an extension of S and that the controller Ψ_e is contractible to the controller Ψ .Moreover, suppose that the controller Ψ is applied to the system S and that the controller Ψ_e is applied to the system S_e .then , stability (respectively asymptotic stability) of the expanded closed-loop system (obtained by applying Ψ_e to S_e) implies the stability (respectively asymptotic stability) of the original closed –loop system (obtained by applying Ψ to S).

The controller (2.30) is contractible to the controller (2.27), if and only if there exist full-rank matrices $T \in \Re^{n_e \times n}$, $N \in \Re^{m \times m_e}$ and $M \in \Re^{p \times p_e}$ such that:

$$\begin{cases} FM = MF \\ GCA^j = MG_e C_e A_e^j T & \forall \text{j}=0,1,2,...n \\ GCA^j BN = MG_e C_e A_e^j B_e & \forall \text{j}=0,1,2,...n \\ HM = NH_e \\ KCA^j = NK_e C_e A_e^j T & \forall \text{j}=0,1,2,...n \\ KCA^j BN = NK_e C_e A_e^j \mathrm{B_e} & \forall \text{j}=0,1,2,...n \end{cases} \tag{2.33}$$

These conditions can be reduced to simpler ones if the extended system is an extension of the original system [30].

- **Theorem (2)**

Given that the system S_e is an extension of the system S, the controller (2.28) for the system S_e is contractible to the controller (2.27) for the system S , if there exists a full rank matrix $M \in \Re^{p \times p_e}$ such that:

$$\begin{cases} FM = MF_e \\ BC = MG_e SC \\ HM = NH_e \\ KC = NK_e SC \end{cases} \tag{2.34}$$

where

$N \in \Re^{m \times m_e}$ and $S \in \Re^{l_e \times l}$ are the matrices satisfying (2.32) and (2.33) [30].

- **Theorem (3)** [30]

If the system S_e is an extension of the system S , then any controller of the form (2.28) for the system S_e is contractible to a controller of the form (2.27) for the system S with

$$\begin{cases} F = F_e \\ G = G_e S \\ H = NH_e \\ K = NK_e S \end{cases} \tag{2.35}$$

II.5.3. State feedback control (S.F.C)

Consider the control laws $u = Kx$ and $u_e = K_e x_e$ for the system S and the extended system S_e respectively. The control law $u_e = K_e x_e$ is contractible to the control law $u = Kx$, if there exist transformation T, N defined by (2.32) such that $Kx(t; x_0, Nu_e) = NK_e x_e(t; Tx_0, u_e)$ $\forall t \geq 0$ for any initial state $x_0 \in \Re^n$ and any input $u_e(t) \in \Re^{m_e}$, $\forall 0 \leq t \leq \infty$ [31].

II.5.4. Sufficient condition for contractibility of S.F.C

A sufficient condition for controllers' contractibility for the case of state feedback control is provided by the following theorem

- **Theorem (4)** [32]

If S_e is an extension of S then any control law of the form (2.30) is contractible to a control law of the form (2.27), and K is given by:

$$K = NK_e T \tag{2.36}$$

Fig.2.4. Structure of gain matrix K [33].

II.6. Optimal overlapping systems

As described in section II.2 an overlapping system is complex system where one or more of its subsystems share a certain number of state variables with other subsystems. To formally introduce the notion of overlapping in optimization, let us consider a linear (linearised) system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ \quad y(t) = Cx(t) \end{cases} \tag{2.37}$$

A **:** $n \times n$ matrix.
B **:** $n \times m$ matrix.
C : $l \times n$ matrix.

In this section, we will focus on two overlapping subsystems. Consider the system S defined by (2.1). Suppose also that the state, input and output are partitioned as

$$\begin{cases} u = \left[u_1^T, u_2^T, u_3^T\right]^T, \quad u_i \in \Re^{m_i}, i = 1,2,3 \\ x = \left[x_1^T, x_2^T, x_3^T\right]^T, \quad x_i \in \Re^{n_i}, i = 1,2,3 \\ y = \left[y_1^T, y_2^T, y_3^T\right]^T, \quad y_i \in \Re^{l_i}, i = 1,2,3 \end{cases} \tag{2.38}$$

where A, B and C has the form

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{2.39}$$

Let us associate a rectangular matrices $T(n_e \times n)$, $N(n \times m_e)$ and $S(l_e \times n)$; where $(n_e \geq n,\ m_e \geq m,\ l_e \geq l)$

$$T = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & I_{n_2} & 0 \\ 0 & I_{n_2} & 0 \\ 0 & 0 & I_{n_3} \end{bmatrix}, \quad N = \begin{bmatrix} I_{m_1} & 0 & 0 & 0 \\ 0 & I_{m_2} & I_{m_2} & 0 \\ 0 & 0 & 0 & I_{m_3} \end{bmatrix}, \quad S = \begin{bmatrix} I_{l_1} & 0 & 0 \\ 0 & I_{l_2} & 0 \\ 0 & I_{l_2} & 0 \\ 0 & 0 & I_{l_3} \end{bmatrix} \tag{2.40}$$

where
I_{n_1}, I_{n_2} and I_{n_3}: represent identity matrices with dimensions corresponding to the subsystems A_{11}, A_{22} and A_{33} respectively.

Now, we can identify the new system as

$$\begin{cases} \dot{x}_e(t) = A_e x_e(t) + B_e u_e(t) \\ \quad y_e(t) = C_e x_e(t) \end{cases} \tag{2.41}$$

where:

$$\begin{cases} A_e = TAT^I + E_A \\ B_e = TBN + E_B \\ C_e = SCT^I + E_C \end{cases} \tag{2.42}$$

This implies that:

$$A_e = \left[\begin{array}{cc|cc} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ \hline A_{11} & 0 & A_{22} & A_{23} \\ A_{11} & 0 & A_{32} & A_{33} \end{array}\right] = \begin{bmatrix} A_{e11} & A_{e12} \\ A_{e21} & A_{e22} \end{bmatrix} \text{ with } E_A = \begin{bmatrix} 0 & \frac{1}{2}A_{12} & -\frac{1}{2}A_{12} & 0 \\ 0 & \frac{1}{2}A_{22} & -\frac{1}{2}A_{22} & 0 \\ 0 & -\frac{1}{2}A_{22} & \frac{1}{2}A_{22} & 0 \\ 0 & -\frac{1}{2}A_{32} & \frac{1}{2}A_{32} & 0 \end{bmatrix} \tag{2.43}$$

with

$$B_e = \left[\begin{array}{cc|cc} B_{11} & B_{12} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{22} & B_{23} \\ \hline B_{21} & B_{22} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{32} & B_{33} \end{array}\right] = \begin{bmatrix} B_{e11} & B_{e12} \\ B_{e21} & B_{e22} \end{bmatrix} \tag{2.44}$$

and $E_B = 0$

$$C_e = \left[\begin{array}{cc|cc} C_{11} & C_{12} & 0 & C_{13} \\ C_{21} & C_{22} & 0 & C_{23} \\ \hline C_{21} & 0 & C_{22} & C_{23} \\ C_{31} & 0 & C_{32} & C_{33} \end{array}\right] = \begin{bmatrix} C_{e11} & C_{e12} \\ C_{e21} & C_{e22} \end{bmatrix} \tag{2.45}$$

with

$$E_C = \begin{bmatrix} 0 & \frac{1}{2}C_{12} & -\frac{1}{2}C_{12} & 0 \\ 0 & \frac{1}{2}C_{22} & -\frac{1}{2}C_{22} & 0 \\ 0 & -\frac{1}{2}C_{22} & \frac{1}{2}C_{22} & 0 \\ 0 & -\frac{1}{2}C_{32} & \frac{1}{2}C_{32} & 0 \end{bmatrix}$$

Let us now assume that the expanded system s_e can be stabilized with a D.C Law $u = K_{ed} x_e$ where

$$K_{ed} = K_d V^{-1} = \left[\begin{array}{cc|cc} K_{e11} & K_{e12} & 0 & 0 \\ \hline 0 & 0 & K_{e23} & K_{e24} \end{array}\right] \tag{2.46}$$

K_{ed} : Represents an overlapping Controller, which is decentralized controller in the expanded space.

Equation (2.46) can be contracted to get the gain matrix of the form below [29]

$$K_d = \left[\begin{array}{cc:c} K_{11} & K_{12} & 0 \\ \hdashline 0 & K_{23} & K_{24} \end{array}\right] \tag{2.47}$$

The inputs and outputs of the expanded system are related to those of the original system S by:

$$u_{e1} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},\; u_{e2} = \begin{bmatrix} u_2 \\ u_3 \end{bmatrix},\; y_{e1} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},\; y_{e2} = \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} \tag{2.48}$$

II.7. Optimal decentralized feedback control

Many of today's technological and social problems involve very complex and large dimensional systems. For such large dimensional systems, it is usually necessary to avoid centralized controllers due to a number of reasons such as the cost of implementation, complexity of on-line computations, complexity of controllers design, and reliability. In such situation, a decentralized overlapping feedback structure may be imposed.

A decentralized controller can be designed as a feedback controller which consists of independent control stations, each of which use the measurements data y_i and influences the control input u_i only of the attached subsystem. The control laws, which describe how u_i has to be chosen for given y_i, are independent of each other. The information flow from the plant through the controller back to the plant is divided into separate flows through the control stations.

Despite considerable efforts and large number of new results in the theory of L.S.Ss, the fundamental problem of optimal D.C has remained unsolved. The simple reason is that the decentralized information structure constraints have not been successfully incorporated in any of the standard optimization frameworks. It should be noted that the optimal decentralized controllers are designed in the expanded system and not in each subsystem.

II.7.1. Optimal output feedback control

The optimal control problem in modern control theory is usually given in the form of minimization of some performance index or a cost function constrained by the state model of the system. In this section we are going to derive necessary and sufficient conditions of optimality for the expanded system and we will recognize that the optimal control designed in the expanded system is contracted to an optimal D.C for the original system.

Let us consider the cost function for the original system

$$J = \int_0^\infty \left(x^T Qx + u^T Ru + z^T Vz + \dot{z}^T W\dot{z} + 2x^T Uz + 2u^T Y\dot{z} \right) dt \tag{2.49}$$

where

Q, R, V, W, U and Y are appropriately dimensioned constant matrices such that

$$\hat{Q} = \begin{bmatrix} Q & U \\ U^T & V \end{bmatrix} \text{ and } \hat{R} = \begin{bmatrix} R & Y \\ Y^T & W \end{bmatrix} \quad (2.50)$$

are respectively symmetric positive semi-definite and symmetric positive definite matrices, x and u are respectively state and the control vectors of the system S, z is the state vector of the controller Ψ.

Also, consider the cost function for the expanded system

$$J_e = \int_0^\infty \left(x_e^T Q_e x_e + u_e^T R_e u_e + z_e^T V_e z_e + \dot{z}_e^T W_e \dot{z}_e + 2x_e^T U_e z_e + 2u_e^T Y_e \dot{z}_e \right) dt \quad (2.51)$$

where

Q_e, R_e, V_e, W_e, U_e and Y_e are appropriately dimensioned constant matrices such that

$$\hat{Q}_e = \begin{bmatrix} Q_e & U_e \\ U_e^T & V_e \end{bmatrix} \quad (2.52)$$

is a symmetric positive semi-definite matrix with:

$$\begin{cases} R_e = blockdiag(R_{e1}, R_{e2}, ..., R_{en}), \ R_{ei} \in \Re^{m_{ei}, m_{ei}}, \ i = 1, 2, ..., n \\ W_e = blockdiag(W_{e1}, W_{e2}, ..., W_{en}), \ W_{ei} \in \Re^{p_{ei}, p_{ei}}, \ i = 1, 2, ..., n \\ Y_e = blockdiag(Y_{e1}, Y_{e2}, ..., Y_{en}), \ Y_{ei} \in \Re^{m_{ei}, p_{ei}}, \ i = 1, 2, ..., n \end{cases}$$

where m_{ei} is the dimension of the i^{th} control input of the system (2.1) and p_{ei} is the dimension of the i^{th} decentralized controller Ψ_{ei}, described in (2.28) and

$$\hat{R}_{ei} = \begin{bmatrix} R_{ei} & Y_{ei} \\ Y_{ei}^T & W_{ei} \end{bmatrix}, \ i = 1, 2, ..., n \quad (2.53)$$

are symmetric positive definite matrices, x_e and u_e are respectively the state and the control vectors of the system S_e, z_e is the state vector of the controller Ψ_e [4]. Now, assume that S_e is an extension of S [30].

A. Initial problem

Our objective is to find F, G, H and K of the controller that minimize J such that the closed loop control system

$$S_c : \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A + BKC & BH \\ GC & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \quad (2.54)$$

is asymptotically stable.

with $$J(F, G, H, K, x_0, z_0) \le J(\bar{F}, \bar{G}, \bar{H}, \bar{K}, x_0, z_0), \ \forall (\bar{F}, \bar{G}, \bar{H}, \bar{K}) \in \Omega \quad (2.55)$$

B. Expanded problem

Find F_e, G_e, H_e and K_e of the controller Ψ_e that minimizes J_e such that the closed loop control system

$$S_{ec}: \begin{bmatrix} \dot{x}_e \\ \dot{z}_e \end{bmatrix} = \begin{bmatrix} A_e + B_e K_e C_e & B_e H_e \\ G_e C_e & F_e \end{bmatrix} \begin{bmatrix} x_e \\ z_e \end{bmatrix} \tag{2.56}$$

is asymptotically stable.
with

$$J_e(F_e, G_e, H_e, K_e, x_{e0}, z_{e0}) \leq J_e(\bar{F}_e, \bar{G}_e, \bar{H}_e, \bar{K}_e, x_{e0}, z_{e0}), \quad \forall (\bar{F}_e, \bar{G}_e, \bar{H}_e, \bar{K}_e) \in \Omega_e \tag{2.57}$$

$J_e(F_e, G_e, H_e, K_e, x_{e0}, z_{e0})$: denotes the value of the cost function (2.51) for the closed-loop system (2.56) with initial state $\begin{bmatrix} x_{e0}^T & z_{e0}^T \end{bmatrix}^T$.

To progress this, we must convert the expressions of the performance indices to a standard expression that is LQR problem to derive the necessary conditions of optimality. Let us introduce the following system

$$\hat{S}_e: \begin{cases} \dot{\hat{x}}_e = \hat{A}_e \hat{x}_e + \sum_{i=1}^{n} \hat{B}_{ei} \hat{u}_{ei} \\ \hat{y}_{ei} = \hat{C}_{ei} \hat{x}_e \end{cases} \quad i = 1, 2, ..., n \tag{2.58}$$

where

$$\begin{cases} \hat{x}_e = \begin{bmatrix} x_e^T, z_{e1}^T, z_{e2}^T, ..., z_{en}^T \end{bmatrix} \\ \hat{u}_{ei} = \begin{bmatrix} u_{ei}^T, \dot{z}_{ei}^T \end{bmatrix}^T, i = 1, 2, ..., n \\ \hat{y}_{ei} = \begin{bmatrix} y_{ei}^T, z_{ei}^T \end{bmatrix}^T, i = 1, 2, ..., n \\ \hat{A}_e = blockdiag(A_e, O_{p_e \times p_e}), i = 1, 2, ..., n \\ \hat{C}_{ei} = blockdiag(C_{ei}, \Delta_i), i = 1, 2, ..., n \end{cases} \tag{2.59}$$

and

$$\Delta_i = \begin{bmatrix} 0_{p_e \times \sum_{j=1}^{i-1} p_{ej}}, I_{p_e}, 0_{p_e \times \sum_{j=i+1}^{\upsilon} p_{ej}} \end{bmatrix}, \quad i = 1, 2, ..., n$$

We define also:

$$\begin{cases} \hat{B}_e = blockdiag(B_e, I_p) \\ \hat{C}_e = blockdiag(C_e, I_p) \end{cases}$$

Now, the closed loop system S_{ec} defined by (2.58) is equivalent to the closed loop system defined by applying to the system $\hat{S}_e$, the decentralized static feedback law defined by

$$\hat{u}_{ei} = \hat{K}_e \hat{y}_{ei}, \; i = 1, 2, ..., n \tag{2.60}$$

where

$$\hat{K}_{ei} = \begin{bmatrix} K_{ei} & H_{ei} \\ G_{ei} & F_{ei} \end{bmatrix}, \; i = 1, 2, ..., n \tag{2.61}$$

Hence, we have converted the expanded problem into finding a stabilizing decentralized feedback gain matrices $\hat{K}_{ei}\,(i=1,2,...,n)$ for the system $\hat{S}_e$ which minimize the cost function

$$\hat{J}_e = J_e = \int_0^\infty (\hat{x}_e^T \hat{Q}_e \hat{x}_e + \sum_{i=1}^{\upsilon} \hat{u}_{ei}^T \hat{R}_{ei} \hat{u}_{ei})dt \tag{2.62}$$

with

$\hat{Q}_e$ and $\hat{R}_{ei}$ are as defined before in (2.52), (2.53).

If we assume that the extended system is stabilizable under the closed loop structure, the optimal cost for the expanded problem is

$$J_e^*(x_{e0}, z_{e0}) = trace(P_e \hat{X}_{e0}) \tag{2.63}$$

where

$$\hat{X}_{e0} = \hat{x}_{e0}\hat{x}_{e0}^T$$
$$\hat{x}_{e0} = \left[x_{e0}^T z_{e0}^T \right]$$

and p_e is the solution of:

$$\Phi_e^T P_e + P_e \Phi_e + \hat{Q}_e + \hat{C}_e^T \hat{K}_e^T \hat{R}_e \hat{K}_e \hat{C}_e = 0 \tag{2.64}$$

where

$$\Phi_e = \hat{A}_e + \sum_{i=1}^{\upsilon} \hat{B}_{ei} \hat{K}_{ei} \hat{C}_{ei} \tag{2.65}$$

$$\hat{K}_e = \begin{bmatrix} K_e & H_e \\ G_e & F_e \end{bmatrix},\ \hat{R}_e = \begin{bmatrix} R_e & Y_e \\ Y_e & W_e \end{bmatrix}$$

Necessary conditions for $\hat{K}_e$ to be an optimal solution is:

$$\frac{\delta}{\delta \hat{K}_{ei}} = trace\left[P_e \hat{X}_{e0} + L_e \Lambda_e \right] \tag{2.66}$$

for $i=1,2,...,n$, where Λ_e is the left-hand side of (2.65) and the Lagrange multiplier matrix L_e satisfies

$$\Phi_e L_e + L_e \Phi_e^T + \hat{X}_{e0} = 0$$

The condition (2.66) can be reduced to:

$$\frac{\delta}{\delta \hat{K}_{ei}} = trace\left[L_e P_e \hat{B}_e \hat{K}_e \hat{C}_e + \frac{1}{2} L_e \hat{C}_e^T \hat{K}_e^T \hat{R}_e \hat{K}_e \hat{C}_e \right] = 0$$

or

$$\frac{\delta}{\delta \hat{K}_{ei}} = trace\left[L_e P_e \hat{B}_{ei} \hat{K}_{ei} \hat{C}_{ei} + \frac{1}{2} L_e \hat{C}_{ei}^T \hat{K}_{ei}^T \hat{R}_{ei} \hat{K}_{ei} \hat{C}_{ei} \right] = 0$$

equivalently

$$\nabla_{K_i} J_e = \hat{B}_{ei}^T P_e L_e \hat{C}_{ei}^T + \hat{R}_{ei} \hat{K}_{ei} \hat{C}_{ei} L_e \hat{C}_{ei}^T = 0 \qquad \text{for } i=1,2,...,n \tag{2.67}$$

Hence, assuming that $\hat{C}_{ei} L_e \hat{C}_{ei}^T$ is invertible, the matrices of the optimal controller are given by:

$$\hat{K}_{ei} = \begin{bmatrix} K_{ei} & H_{ei} \\ G_{ei} & F_{ei} \end{bmatrix} = -\hat{R}_{ei}^{-1} \hat{B}_{ei}^T P_e L_e \hat{C}_{ei}^T (\hat{C}_{ei} L_e \hat{C}_{ei}^T)^{-1},\ \ i=1,2,...,n \tag{2.68}$$

Where the Lagrange multiplier matrix L_e satisfies

$$\Phi_e L_e + L_e \Phi_e^T + \hat{X}_{e0} = 0 \tag{2.69}$$

Therefore, the optimal expanded controller can be found by solving simultaneously the three nonlinear equations (2.64), (2.67) and (2.69) [4]
For static controllers the previous equations is reduced to:

$$\begin{cases} \Phi^T P + P\Phi + Q + C^T K^T RKC = 0 \\ \nabla_{K_i} J = B_i^T PLC_i^T + R_i K_i C_i LC_i^T = 0 \\ K_i = -R_i^T B_i^T PLC_i^T (C_i LC_i^T)^{-1} \\ \Phi L + L\Phi^T + X_0 = 0 \end{cases} \tag{2.70}$$

where

$$\Phi = A + \sum_{i=1}^{n} B_i K_i C_i \tag{2.71}$$

Since in many applications $X(0)$ may not be known, we usually do not minimize the P.I but its expected value:

$$E\{J\} = \frac{1}{2} E\left\{\hat{x}_e^T(0)\ P\ \hat{x}_e(0)\right\} = trace(P_e \hat{X}_{e0}) \tag{2.72}$$

where the symmetric matrix $\hat{X}_{e0}$ is equal to:

$$\hat{X}_{e0} = E\left\{\hat{x}_e^T(0)\ \hat{x}_e(0)\right\} \tag{2.73}$$

which is the initial autocorrelation of the initial state. Furthermore, we assume that the initial states are uniformly distributed on the unit spheres, then $\hat{X}_0 = I$ (the identity). By solving these equations we obtain the optimal controller Ψ_e which satisfies the necessary condition of optimality for the extended system. The following theorem shows that this controller can be contracted to get the controller Ψ that satisfies the necessary conditions of optimality for the original system.

- **Theorem (5)**

Let S_e be an extension of S and T, N and S to be the transformations satisfying the conditions of extension. Suppose that the controller Ψ_e described in (2.30), stabilized the system S_e and that the matrices F_e, G_e, H_e and K_e of Ψ_e satisfy the necessary conditions of optimality for the initial state vectors $x_{e0} = Tx_0$ and $z_{e0} = z_0$ for some $x_0 \in \Re^n$ and $z_0 \in \Re^p$.
suppose that $rank(N[K_e MH_e]) = rank([K_e MH_e])$

where

$$\begin{cases} M_e = blockdiag(M, I_p) \\ N_e = blockdiag(N, I_p) \end{cases}$$

$\hat{K}_e, \hat{R}_e$: are as defined in eq (2.65) and $\hat{R}$ as defined in eq (2.50), also let the remaining weighting matrices.
Q, U and V be chosen such that:

$$\begin{cases} U = T^T U_e \\ Q = T^T Q_e T \\ V = V_e \end{cases}$$

then,

$$\begin{cases} F = F_e \\ G = G_e S \\ H = NH_e \\ K = NK_e S \end{cases}$$

stabilizes the system S, the matrices F, G, H and K of Ψ satisfy the necessary conditions of optimality for the original problem with initial state x_0 and z_0, and

$$J = (F, G, H, K, x_0, z_0) = J_e(F_e, G_e, H_e, K_e, x_{e0}, z_{e0}) \qquad (2.74)$$

II.7.2. Optimal state feedback control

- **Definition 1**

Let K and K_e be such that $A + BK$ and $A_e + B_e K_e$ are asymptotically stable, then the pair (S_e, J_e) includes the pair (S, J) with respect to the feedback control laws $(Kx, K_e x_e)$ if there exist transformations $T: \Re^n \rightarrow \Re^{n_e}$; $N: \Re^{m_e} \rightarrow \Re^m$ such that S_e includes S, $K_e x_e$ is contractible to Kx and:

$$J(x_0, Kx) = J_e(Tx_0, K_e x_e)$$

for any initial state $x_0 \in \Re^n$.
The matrices of J and J_e can be related as

$$\begin{cases} Q_e = (T^I)^T Q T^I + M_Q \\ R_e = N^T R U + N_R \end{cases} \qquad (2.75)$$

where M_Q and N_R are constant complementary matrices [8].

- **Theorem (6)** [34]

If S_e is an extension of $S, K = NK_e T$, and K_e is such that: $A_e + B_e K_e$ is asymptotically stable, then the pair (S_e, J_e) includes the pair (S, J) with respect to $(Kx, K_e x_e)$ if and only if:

$$T^T (M_Q + K_e^T N_R N_e) T = 0 \qquad (2.76)$$

II.7.3. Overlapping decentralized optimal feedback controller

The decentralized controllers of the system S_e are:

$$\begin{cases} \Psi_{e1}: \begin{cases} \dot{z}_{e1} = F_{e1} z_{e1} + \begin{bmatrix} G_{e1}^1 & G_{e2}^1 \end{bmatrix} y_{e1} \\ u_{e1} = \begin{bmatrix} H_{e1}^1 \\ H_{e2}^1 \end{bmatrix} z_{e1} + \begin{bmatrix} K_{11}^1 & K_{12}^1 \\ K_{21}^1 & K_{22}^1 \end{bmatrix} y_{e1} \end{cases} \\ \Psi_{e2}: \begin{cases} \dot{z}_{e2} = F_{e2} z_{e2} + \begin{bmatrix} G_{e1}^2 & G_{e2}^2 \end{bmatrix} y_{e2} \\ u_{e2} = \begin{bmatrix} H_{e1}^2 \\ H_{e2}^2 \end{bmatrix} z_{e2} + \begin{bmatrix} K_{11}^2 & K_{12}^2 \\ K_{21}^2 & K_{22}^2 \end{bmatrix} y_{e2} \end{cases} \end{cases} \qquad (2.77)$$

These decentralized controllers can be now contracted for implementation on the original system to the controller Ψ defined by

$$\Psi : \begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} F_{e1} & 0 \\ 0 & F_{e2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} G_{e1}^1 & G_{e2}^1 & 0 \\ 0 & G_{e1}^2 & G_{e2}^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \\ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} H_{e1}^1 & 0 \\ H_{e2}^1 & H_{e1}^2 \\ 0 & H_{e2}^2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} K_{e11}^1 & K_{e12}^1 & 0 \\ K_{e21}^1 & \frac{1}{2}(K_{e22}^1 + K_{e11}^2) & K_{e12}^2 \\ 0 & K_{e21}^2 & K_{e22}^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \end{cases} \tag{2.78}$$

This satisfies the information structure constraints imposed by the two overlapping subsystems s_1 and s_2 .
From the previous theorem we note that if the controller achieves asymptotic stability and satisfies the optimality conditions , then the contracted controller achieves asymptotic stability and satisfies the optimality conditions , furthermore $J = J_e$ [16].

II.7.4. Overlapping optimal state feedback controller

Consider the system S defined by (2.10), now, suppose that the control laws $u_{e1} = K_e^1 x_{e1}$, $u_{e2} = K_e^2 x_{e2}$ are design to minimize the cost function

$$J_{ei} = \int_0^\infty (x_i^T Q_{ei} x_{ei} + u_{ei}^T R_{ei} u_{ei}) dt, \quad i = 1, 2 \tag{2.79}$$

where
$$K_{ei} = -R_{ei}^{-1} B^T P_{ei}, \quad i = 1, 2 \tag{2.80}$$
and P_{ei} is the positive semi definite solution of

$$A_{ei}^T P_{ei} + P_{ei} A_{ei} + Q_{ei} - P_{ei} B_{ei} R_{ei}^{-1} B_{ei}^T P_{ei} = 0$$
(2.81)
which is Ricatti equation [8].
The total cost function for the expanded system is

$$J_e = J_{e1} + J_{e2} = \int_0^\infty (x_e^T Q_e x_e + u_e^T R_e u_e) dt \tag{2.82}$$

where

$$\begin{cases} Q_e = blockdiag(Q_{e1}, Q_{e2}) \\ R_e = blockdiag(R_{e1}, R_{e2}) \end{cases} \tag{2.83}$$

If we write $K_e^i = \begin{bmatrix} K_{e11}^i & K_{e12}^i \\ K_{e12}^i & K_{e22}^i \end{bmatrix}$, The feedback control law $u_e = K_e x_e, \quad \mathrm{K_e} = block(K_e^1, K_e^2)$ can be contracted to the original space as $u = Kx$ Where

$$K = \left[\begin{array}{c:c:c} K_{e11}^1 & K_{e12}^1 & 0 \\ \hdashline K_{e21}^1 & K_{e22}^1 + K_{e11}^2 & K_{e12}^2 \\ \hdashline 0 & K_{e21}^2 & K_{e22}^2 \end{array}\right] \tag{2.84}$$

II.8. Summary

In this chapter, we defined the overlapping decomposition and its mathematical background, and then we moved to the formulation of optimal control problem for different performance index, after that we have defined the optimal overlapping control technique and finally we combined the last technique with decentralized control structure to get decentralized optimal overlapping decomposition control.

We concluded that it is possible to apply this technique for real physical large scale systems for predefined condition.

Chapter three

Application of Optimal Decentralized Overlapping Controller Design

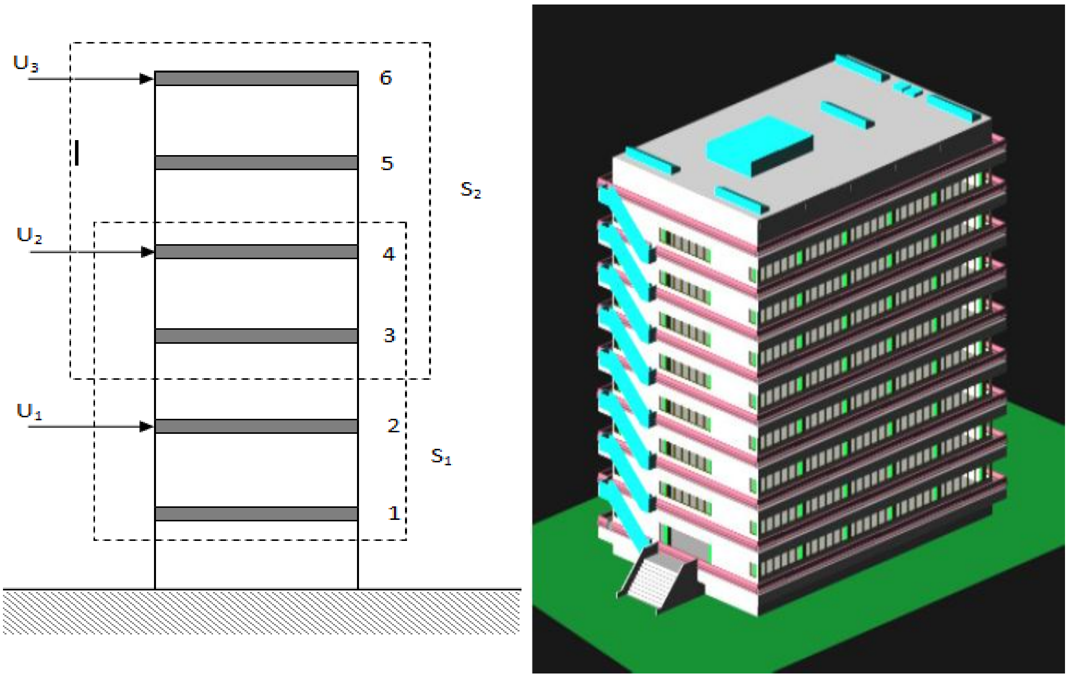
In this chapter, we present examples for which optimal decentralized overlapping decomposition technique given in the previous chapter is realizable. The first example is six floor building system with two degree of freedom. The second is web winding system with three motors and also two degree of freedom.

The same algorithm has been applied for the two examples.

III.1.First application: Building overlapping structure system

Engineering is a broad term that includes many disciplines, all of which have in common the application of science for practical purposes. “Applied science” is approximately equivalent with “engineering.” While people have built many things that today are within the scope of civil engineering – buildings, dams, canals, roads, bridges – it is the scientific aspect to the design and building of such construction that defines modern civil engineering. People combined a practical knowledge of materials and construction with the mathematics and science that were then available [37].

Consider the mechanical second order building system shown in Figure.III.1



FigIII.1. Overlapping structure of building system.

The system shown in FigIII.1 can be represented by the following mathematical model [36]

$$\begin{cases} M\ddot{q} + D\dot{q} + Sq = Bu \\ y = Cq \\ v = V\dot{q} \end{cases} \tag{3.1}$$

where

$M:6\times6$: is the mass matrix, symmetric, positive definite matrix.

$D:6\times6$: is the damping matrix.

$S:6\times6$: is the stiffness matrix.

$q:6\times1$: is the displacement vector, represents the degree of freedom of the system.

$u:3\times1$: is the input signal, sinusoidal signal in this example.

$B:6\times3$: is the input matrix, represents locations of actuators.

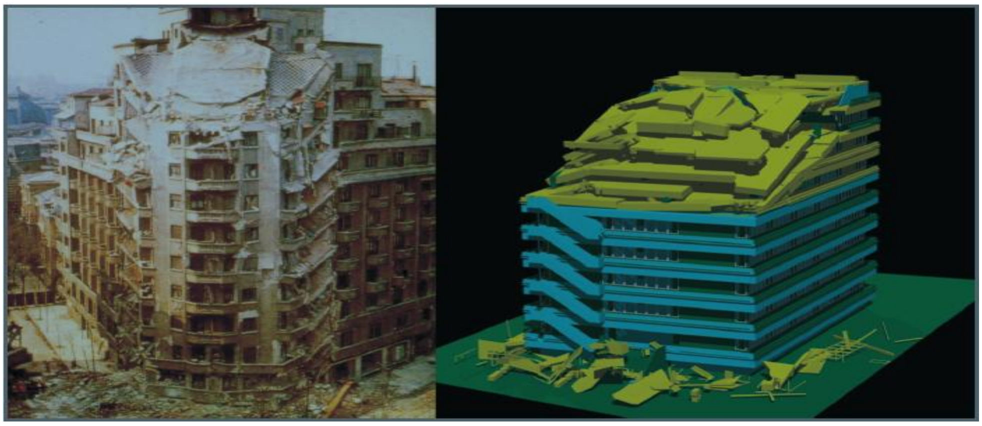


Fig.III.2. Failure response of a building system to earthquake disturbance.

Suppose that this system can be written as

$$\begin{cases} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \\ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V_{11} & 0 & 0 \\ 0 & V_{22} & 0 \\ 0 & 0 & V_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \end{cases} \tag{3.2}$$

Where dashed lines indicate the subsystems, in this example we have only two subsystem S_1 and S_2 that shares common informations ($M_{22}, D_{22}, S_{22}, B_{22}, C_{22}, V_{22}$)

III.1.1.Expansion and contraction

We have the overlapping system (3.2) and we want to transform it into non-overlapping system described by

$$s_e : \begin{cases} M_e \ddot{q}_e + D_e \dot{q}_e + S_e q_e = B_e u_e \\ y_e = C_e q_e \\ v_e = V_e \dot{q}_e \end{cases} \tag{3.3}$$

To do that, we consider the transformation matrices between systems (3.2) and (3.3)

$$\begin{cases} q_e = Tq \\ u_e = U^I u \\ y_e = Gy \\ v_e = Hv \end{cases} \tag{3.4}$$

or
$$\begin{cases} q = T^I q_e \\ u = Uu_e \\ y = G^I y_e \\ v = H^I v_e \end{cases} \tag{3.5}$$

where $T^I T = I_n = I_6$, $UU^I = I_m = I_3$, $G^I G = I_p = I_6$, $H^T H = I_r = I_6$ And T^I, U^I, G^I, H^I indicates the pseudo-inverse of T, U, G, H respectively.

III.1.2. Sufficient condition for expansion

The system (3.3) is an expansion of the system (3.1) (or the system (3.1) is contraction of the system (3.3)) if there exist transformation T, U, G and H satisfying (3.4) such that for any initial state $(q_e(0), \dot{q}_e(0))$ and for any input $u_e(t) \in R^m$ for all $t \geq 0$ [35]

$$\begin{cases} q_e(0) = Tq(0) \\ \dot{q}_e(0) = T\dot{q}(0) \\ u(t) = Uu_e(t) \end{cases} \Rightarrow \begin{cases} q_e(t) = Tq(t) \\ \dot{q}_e(t) = T\dot{q}(t) \\ v_e(t) = Hv(t) \end{cases} \tag{3.6}$$

III.1.3.Methods for deriving the extension's condition

Essentially there exist two (02) methods to derive the condition of extension:

- **Method one**

Requires working directly with the matrix second order equation in both original and extended system, it means we need to use the matrices M and M_e.

- **Method two**

Starts by transforming the second order system into an equivalent first order system which means it requires working with M^{-1} and M_e^{-1}.

III.1.4.Transformation into state space model

Consider the system (3.1) and its expansion (3.3); define the state vectors x, x_e as: $x = (q^T, \dot{q}^T)^T$, $x_e = (q_e^T, \dot{q}_e^T)^T$, then (3.1) and (3.3) can be written as

$$s_x : \begin{cases} \dot{x} = A_x x + B_x u \\ y_x = C_x x \end{cases} \tag{3.7}$$

$$s_{ex} : \begin{cases} \dot{x}_e = A_{ex} x_e + B_{ex} u_e \\ y_{ex} = C_{ex} x_e \end{cases} \tag{3.8}$$

where

$$A_x = \begin{bmatrix} 0_{6\times6} & I_6 \\ -M^{-1}S & -M^{-1}D \end{bmatrix} ,\ B_x = \begin{bmatrix} 0_{6\times3} \\ M^{-1}B \end{bmatrix},\ C_x = diag(C,V)$$

and

$$A_{ex} = \begin{bmatrix} 0_{8\times8} & I_8 \\ -M_e^{-1}S_e & -M_e^{-1}D_e \end{bmatrix},\ B_{ex} = \begin{bmatrix} 0_{8\times4} \\ M_e^{-1}B_e \end{bmatrix}, C_{ex} = diag(C_e, V_e)$$

Consider the transformation T, U, G and H satisfying (3.6) for the original system; define the transform for the expanded system as

$$T_d = diag(T,T)\ ;\ C_d = diag(G,H)$$

This implies that

$$\begin{cases} x_e(0) = T_d x(0) \\ u(t) = Uu_e(t) \end{cases} \Rightarrow \begin{cases} x_e(t) = T_d x(t) \\ y_{ex}(t) = C_d x(t) \end{cases} \tag{3.9}$$

- **Theorem one**

The system s_e is an extension of the system s or equivalently s is disextension of s_e if and only if there exists full rank transformation matrices T, U, G and H such that

$$\begin{cases} M_e^{-1}S_eT = TM^{-1}S \\ M_e^{-1}D_eT = TM^{-1}D \\ M_e^{-1}B_e = TM^{-1}BU \\ GC = C_eT \\ HV = V_eT \end{cases} \tag{3.10}$$

This equation are found by transforming the system (3.1) and (3.3) into state space model. Equa (3.10) can be written as

$$\begin{cases} M_e^{-1} = TM^{-1}T^I + M_{cq} \\ S_e = TST^I + S_{cq} \\ D_e = TDT^I + D_{cq} \\ B_e = TBU + B_{cq} \\ C_e = GCT^I + C_c \\ V_e = HVT^I + V_c \end{cases} \tag{3.11}$$

where

$$M_e = \left[\begin{array}{cc|cc} M_{11} & M_{12} & 0 & M_{13} \\ M_{21} & M_{22} & 0 & M_{23} \\ \hline M_{21} & 0 & M_{22} & M_{23} \\ M_{31} & 0 & M_{32} & M_{33} \end{array}\right],\ D_e = \left[\begin{array}{cc|cc} D_{11} & D_{12} & 0 & D_{13} \\ D_{21} & D_{22} & 0 & D_{23} \\ \hline D_{21} & 0 & D_{22} & D_{23} \\ D_{31} & 0 & D_{32} & D_{33} \end{array}\right] \text{ and } S_e = \left[\begin{array}{cc|cc} S_{11} & S_{12} & 0 & S_{13} \\ S_{21} & S_{22} & 0 & S_{23} \\ \hline S_{21} & 0 & S_{22} & S_{23} \\ S_{31} & 0 & S_{32} & S_{33} \end{array}\right]$$

$$B_e = \left[\begin{array}{cc|cc} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & B_{22} & 0 \\ \hline 0 & B_{22} & B_{22} & 0 \\ 0 & 0 & 0 & B_{33} \end{array}\right]$$

$$\begin{cases} F = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix} \\ L = \begin{bmatrix} K_{13}^1 & K_{14}^1 & 0 \\ K_{23}^1 & K_{24}^1 + K_{13}^2 & K_{14}^2 \\ 0 & K_{23}^2 & K_{24}^2 \end{bmatrix} \end{cases}, C_e = \left[\begin{array}{cc|cc} C_{11} & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 \\ \hline 0 & 0 & C_{22} & 0 \\ 0 & 0 & 0 & C_{33} \end{array}\right] \text{ and } V_e = \left[\begin{array}{cc|cc} V_{11} & 0 & 0 & 0 \\ 0 & V_{22} & 0 & 0 \\ \hline 0 & 0 & V_{22} & 0 \\ 0 & 0 & 0 & V_{33} \end{array}\right]$$

where $M_{qc}, S_{qc}, D_{qc}, B_{qc}, C_{qc}$ and V_{qc} are complementary matrices defined in such way to satisfy the condition of extension [27].

- **Theorem two** [27]

The system (3.3) is an expansion of the system (3.1) if

$$\begin{cases} M_{qc}T = 0 \\ K_{qc}T = 0 \\ D_{qc}T = 0 \\ B_{qc} = 0 \\ C_{qc}T = 0 \\ V_{qc}T = 0 \end{cases} \tag{3.12}$$

Equa (3.11) is satisfied by choosing the complementary matrices as

$$[.]_{qc} = \begin{bmatrix} 0 & 0.5[.]_{12} & -0.5[.]_{12} & 0 \\ 0 & 0.5[.]_{22} & -0.5[.]_{22} & 0 \\ 0 & -0.5[.]_{22} & 0.5[.]_{22} & 0 \\ 0 & -0.5[.]_{32} & 0.5[.]_{32} & 0 \end{bmatrix} \tag{3.13}$$

III.1.5. Contractibility of controllers

Consider the static feedback control laws for the overlapping system

$$u = Fy + Lv + w \tag{3.14}$$

and consider the static feedback laws for the expanded system

$$u_e = F_e y_e + L_e v_e + w_e \tag{3.15}$$

w and w_e are external inputs.

- **Theorem three**

The control law (3.14) is contractible to the control law (3.13) if and only if

$$\begin{cases} FC = UF_e GC \\ LV = UL_e HV \end{cases} \tag{3.16}$$

- **Theorem four**

If the system (3.3) is an extension of the system (3.1) and if (3.3) is stable (respectively asymptotically stable) then (3.1) is stable (respectively asymptotically stable) [16].

III.1.6. Decentralized optimal output feedback control

Consider the system (3.7); our goal is to find control law $u = Ky_x$ to minimize the cost function

$$J = \int_{-\infty}^{+\infty} (x^T Qx + u^T Ru)dt \tag{3.17}$$

such that the closed loop system

$$S_c = \begin{cases} \dot{x} = (A + B_x KC_x)x \\ y_x = C_x x \end{cases} \tag{3.18}$$

is asymptotically stable.

III.1.7.Problem solution

First we have the following two subsystems that have been extracted from the expanded system

$$\begin{cases} M_1\ddot{q}_{e1} + D_1\dot{q}_{e1} + S_1 q_{e1} = B_1 u_{e1} \\ y_{e1} = C_1 q_{e1} \\ v_{e1} = V_1 \dot{q}_{e1} \end{cases} \tag{3.19.a}$$

$$\begin{cases} M_2\ddot{q}_{e2} + D_2\dot{q}_{e2} + S_2 q_{e2} = B_2 u_{e2} \\ y_{e2} = C_2 q_{e2} \\ v_{e2} = V_2 \dot{q}_{e2} \end{cases} \tag{3.19.b}$$

Let us transform these two subsystems into state space form to get

$$s_1 : \begin{cases} \dot{x}_{e1} = A_{ex1}x_{e1} + B_{ex1}u_{e1} \\ y_1 = C_{ex1}x_{e1} \end{cases} \tag{3.20.a}$$

$$s_2 : \begin{cases} \dot{x}_{e2} = A_{ex2}x_{e2} + B_{ex1}u_{e2} \\ y_1 = C_{ex2}x_{e2} \end{cases} \tag{3.20.b}$$

where

$$A_{ex1}=\begin{bmatrix}0_{(2+2)\times(2+2)} & I_{(2+2)\times(2+2)}\\ -M_1^{-1}S_1 & -M_1^{-1}D_1\end{bmatrix},\quad B_{ex1}=\begin{bmatrix}0_{(2+2)\times(2+2)}\\ -M_1^{-1}B_1\end{bmatrix},\ C_{ex1}=diag(C_1,V_1)$$

$$A_{ex2}=\begin{bmatrix}0_{(2+2)\times(2+2)} & I_{(2+2)\times(2+2)}\\ -M_2^{-1}S_2 & -M_2^{-1}D_2\end{bmatrix},\quad B_{ex2}=\begin{bmatrix}0_{(2+2)\times(2+2)}\\ -M_2^{-1}B_2\end{bmatrix},\ C_{ex2}=diag(C_2,V_2)$$

We generate the optimal output feedback for each subsystem as $u_i=K_i y_i; i=1,2$, To each subsystem we associate the performance index:

$$J_i=\int_{-\infty}^{+\infty}(x_i^T Q_i x_i+u_i^T R_i u_i)dt \qquad i=1,2 \tag{3.21}$$

The necessary and sufficient conditions of optimality for each subsystem are:

$$\begin{cases}\phi_i^T P_i+P_i\phi_i+Q_i+C_{xi}^T K_i^T R_i K_i C_{xi}=0\\ K_i=-R_i^{-1}B_{xi}^T P_i L_i C_{xi}^T(C_{xi}L_i C_{xi}^T)^{-1}\\ \phi_i L_i+L_i\phi_i^T+X_{0i}=0\end{cases} \tag{3.22}$$

where $\phi_i=A_i+B_{xi}K_iC_{xi}$, $X_{0i}=x_{0i}x_{0i}^T$; generally we take $x_{0i}=I$

The optimal cost can be found as: $J_i=.5*trace(P_iX_{0i})$

and the optimal control law as $u_i=K_i y_i$ where $K_i=\begin{bmatrix}K_{11}^i & K_{12}^i & K_{13}^i & K_{14}^i\\ K_{21}^i & K_{22}^i & K_{23}^i & K_{24}^i\end{bmatrix}$

The control law for expanded system is

$$K_i=\begin{bmatrix}K_{11}^1 & K_{12}^1 & 0 & 0 & K_{13}^1 & K_{14}^1 & 0 & 0\\ K_{21}^1 & K_{22}^1 & 0 & 0 & K_{23}^1 & K_{24}^1 & 0 & 0\\ 0 & 0 & K_{11}^2 & K_{12}^2 & 0 & 0 & K_{13}^2 & K_{14}^2\\ 0 & 0 & K_{21}^2 & K_{22}^2 & 0 & 0 & K_{23}^2 & K_{24}^2\end{bmatrix}$$

and the contracted control law for the original system is:

$$K=\begin{bmatrix}K_{11}^1 & K_{12}^1 & 0 & K_{13}^1 & K_{14}^1 & 0\\ K_{21}^1 & K_{22}^1+K_{11}^2 & K_{12}^2 & K_{21}^1 & K_{24}^1+K_{13}^2 & K_{14}^2\\ 0 & K_{21}^2 & K_{22}^2 & 0 & K_{23}^2 & K_{24}^2\end{bmatrix} \tag{3.23}$$

To apply this control law for the original mechanical system, we must write it in the form:

$$u=Fy+Lv+w$$

Where w is the external input to the mechanical system [39].

We have

$$K=[F,L] \text{ and } K_e=[F_e,L_e] \text{ with } F=\begin{bmatrix}K_{11}^1 & K_{12}^1 & 0\\ K_{21}^1 & K_{22}^1+K_{11}^2 & K_{12}^2\\ 0 & K_{21}^2 & K_{22}^2\end{bmatrix},\ L=\begin{bmatrix}K_{13}^1 & K_{14}^1 & 0\\ K_{23}^1 & K_{24}^1+K_{13}^2 & K_{14}^2\\ 0 & K_{23}^2 & K_{24}^2\end{bmatrix}$$

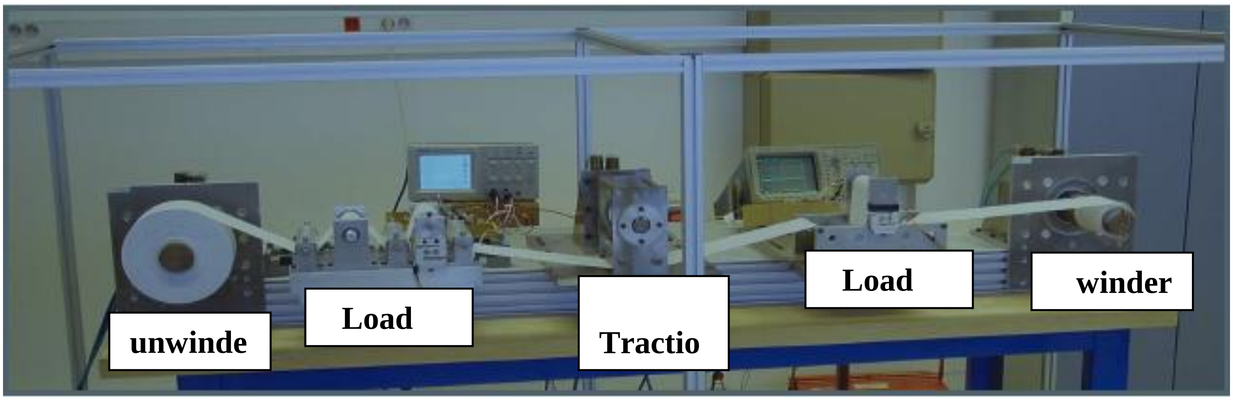
Now, the projection of this control law onto the original mechanical system gives:

$$\begin{cases}M\ddot{q}+(D+BLV)\dot{q}+(K+BFC)q=Bw\\ y=Cq\\ v=V\dot{q}\end{cases} \tag{3.24}$$

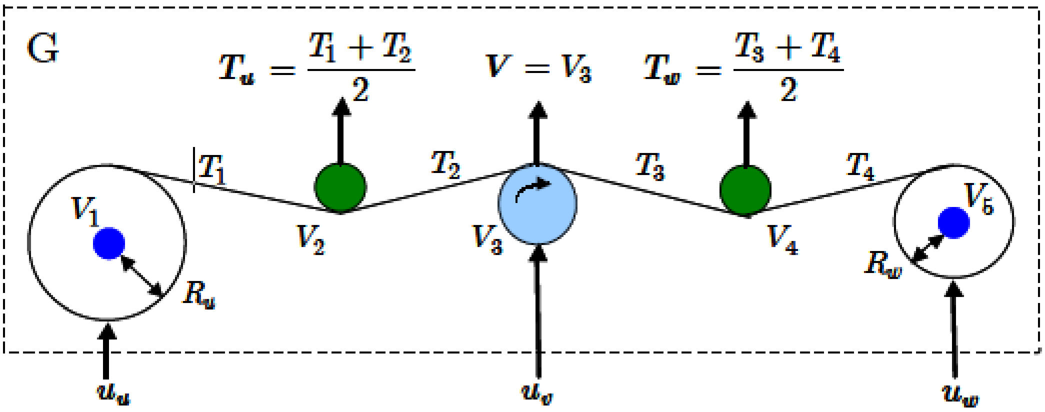
III.2. Application two: Web winding system (W.W.S)

The system under study is quite common in industry. The system has at least three motors (FigureIII.3 (a)): an unwinder, traction motor and a winder, and it present the inherent difficulties of web transport systems [41].

Consider the W.W.S shown in Fig.III.3 (a-[41],b-[39])



-a-



-b-

Fig.III.3. web winding system of three motors

III.2.1.Winding process

The system under study has three motors (FigIII.3 (a)) and exhibits the inherent difficulties of elastic web transport systems.

Figure III.3 (b) shows the different variables used in the model [20,21 and 40].

- The inputs: the control signals u_u, u_v and u_w
- The outputs: unwinding web tension T_u.
  Traction motor's linear velocity V .
  Winding web tension T_w .
- The web velocity is imposed by the traction motor and the web tension is controlled by the unwinding and winding motors.

The mathematical model of three (3) motors web winding system can be described as follows:

First the value of the web speed is bounded by two values $V_{\max}, V_{\min}$.

Let us define epsilon ε as follows

$$\varepsilon = \frac{L_s - L_0}{L_0} = \frac{\Delta L}{L_0} \tag{3.25}$$

ε is the strain of the web.
L is the length of the web; L_s is the stretched length; L_0 is the un-stretched length.

III.2.2.Expression of the tension

- **Hooke's law:**

$$T = ES\varepsilon = ES\frac{L_s - L_0}{L_0} \tag{3.26}$$

where
E : is the Young's modulus of elasticity of the web.
S : The cross section area of the web span.

- **Conservation law of mass**

The mass of the web remains constant between the state without stress and the state under stress

$$\rho_0 L\omega_0 h_0 = \rho_s L\omega_s h_s \Rightarrow \rho_0 S_0 = \rho_s S_s (1+\varepsilon)$$
$$\Rightarrow \frac{m_s}{m_0} = \frac{\rho_s S_s}{\rho_0 S_0} = \frac{1}{1+\varepsilon} \tag{3.27}$$

where
ρ : is the density of the span; ρ_s under stretched, ρ_0 before stretched.
m : is the mass of the web; m_s under stretched, m_0 before stretched [42].
Generally, it is acceptable to assume that $S_s \cong S_0$, so that (3.27) will be

$$\frac{\rho_s}{\rho_0} = \frac{1}{1+\varepsilon} \tag{3.28}$$

- **law of continuity:**

$$\frac{\delta\rho}{\delta t} + \frac{\delta(\rho V)}{\delta t} = 0 \tag{3.29}$$

$$dm = \rho_s S_s = \rho_0 S_0 \Rightarrow \frac{dm}{dt} = \frac{d}{dx}\left(\int_{x_1}^{x_2} \rho(x,t)S(x,t)dx\right)$$
$$= \rho_2(t)S_2(t)v_2(t) - \rho_1(t)S_1(t)v_1(t) \tag{3.30}$$

- **Assumption**

$$\begin{cases} \varepsilon_1(x,t) = \varepsilon_1(t) \Rightarrow \rho(x,t) = \rho_1(t) \\ S(x,t) = S_1(t) \end{cases} \tag{3.31}$$

From the equation above:

$$L\frac{d}{dx}\left[\frac{1}{1+\varepsilon_1(t)}\right] = \frac{v_2(t)}{1+\varepsilon_2(t)} - \frac{v_1(t)}{1+\varepsilon_1(t)} \tag{3.32}$$

Now by approximating

$$\frac{1}{1+\varepsilon} \cong 1-\varepsilon \Rightarrow \dot{\varepsilon}_1(t) = \frac{1}{L}\left[\varepsilon_2(t)v_2(t) - \varepsilon_1(t)v_1(t) + v_1(t) - v_2(t)\right]$$
$$\Rightarrow \dot{T}_1 = \frac{1}{L}\left[-v_1 T_1 + v_2 T_2 + SE(v_1 - v_2)\right] \tag{3.33}$$

For general case i^{th} spam

$$L_i\dot{T}_i = ES(v_i - v_{i-1}) + T_{i-1}v_{i-1} - T_i(2v_{i-1} - v_i) \quad (3.34)$$

III.2.3.Expression of the velocity

The velocity of the web span is equal to the tangential velocity of the roller which means that there is no slide between the roller and web

$$\frac{d}{dt}(J_i\Omega_i) = R_i(T_{i+1} - T_i) + K_iU_i + C_f \quad (3.35)$$

where

$\Omega_i = \dfrac{v_i}{R_i}$: is the angular velocity of the roller i.

K_iU_i : is the input torque.

C_f : is the torque resulted from the total friction coefficient.

J_i : is the moment of inertia of roller i.

R_i : is the radius of the roller i [43].

The variation of R_i is given by the following equation:

$$\frac{dR_i(t)}{dt} = -\frac{hv_i(t)}{2\pi R_i(t)} \quad (3.36)$$

with

h : is the thickness of the web.

The moments of inertia J_i can be calculated as follows

$$J_i(t) = J_{ai} + \frac{\pi\rho L_a}{2}(R_i^4(t) - R_{ai}^4) \quad (3.37)$$

L_a : is the width of the web.

J_{ai}, R_{ai} are the moment of inertia and radius of the motor i.

and $$F_i = \beta_{fi}\Omega_i \quad (3.38)$$

with $$\Omega_i = \frac{v_i}{R_i(t)}$$

β_{fi} : is the viscosity friction constant for motor i.

We can find a linear model of this system by linearizing these equations around the nominal velocity and tension (v_0, T_0)

The equation (3.30) will be

$$L_i\dot{T}_i = (ES + T_0)(v_i - v_{i-1}) + v_0(T_{i-1} - T_i) \quad (3.39)$$
$$L\dot{T}_2 = (ES + T_0)(v_2 - v_1) + v_0(T_1 - T_2)$$

Now, state space representation of this system is as follows:

$$\begin{cases} E(t)\dot{X} = A(t)X + B(t)U \\ Y = CX + DU \end{cases} \quad (3.40)$$

where

$$\begin{cases} X^T = (v_1\ T_1\ v_2\ T_2\ v_3\ T_3\ v_4\ T_4\ v_5) \\ U^T = (U_u\ U_v\ U_w) \\ Y^T = (T_u\ v_3\ T_w) \end{cases} \quad (3.41)$$

and

$$A(t)=\begin{bmatrix} -f_u(t) & R_u^2(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_0 & -v_0 & E_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -R_2^2 & f_2 & R_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_0 & -E_0 & -v_0 & E_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_f^2 & -f_t & R_f^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_0 & -E_0 & -v_0 & E_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -R_4^2 & -f_4 & R_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & v_0 & -E_0 & -v_0 & E_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_w^2(t) & f_w(t) \end{bmatrix}$$

$$B(t)=\begin{bmatrix} -K_u R_u(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & K_t R_t & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_w R_w(t) \end{bmatrix}, \quad \text{and} \qquad C(t)=\begin{bmatrix} 0 & .5 & 0 & .5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .5 & 0 & .5 & 0 \end{bmatrix}$$

$$E(t)=\begin{bmatrix} J_u(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & v_0 & 0 & L_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_w(t) \end{bmatrix}$$

and

$$R_w(t)=\sqrt{R_u^2(0)+R_w^2(0)-R_u^2(t)} \qquad (3.42)$$

The system described above is time varying system because some of its parameters are varying with time [43].

- **Assumption**

Because the parameters of the system described above are varying very slowly with time we can consider it constant in time interval $\Delta t=\left[t_0 \quad t_{\text{simulation}}\right]$, so that the system is LTI in the simulation interval.

Now let us introduce the parameters of the system described above

Parameter	Its definition	Its value
L	Length of the web between unwinding and winding roller	1.9 m
L_1	Length of web between unwinding and roller 2	0.475 m
L_2	Length of web between roller 2 and roller 3	0.475 m
L_3	Length of web between roller 3 and roller 4	0.475 m

L_4	Length of web between roller 4 and winding roller	0.475 m
T_0	Nominal tension of the web	1.5 Kg(1Kg=10N)
E	Young' elasticity coefficient	0.16×10^9 N/m^2
L_a	The width of the web	0.275×10^{-3} m
h	the thickness of the web	0.1 m
K_u	The torque of the unwinding roller	6.8 Nm
K_w	The torque of the winding roller	6.8 Nm
K_t	The torque of the roller 3	2.6 Nm
Ω_u	Angular velocity of the unwinding roller	300 rpm
Ω_w	Angular velocity of the winding roller	300 rpm
Ω_t	Angular velocity of the roller 3	300 rpm
$\beta_{fu}=\beta_{ft}=\beta_{fw}$	Viscosity friction coefficients of unwinding, tractor and winding motors	$5.9828*e^{-5} N\ m\ \sec/rad$

Tab.III.1. Parameters used for simulation of three motors web winding system.

Here it is some values of $R_u(t)$ for different values of time

Tab.III.2. Values taken for time varying parameters.

$R_u(t)$	Interval time	$R_w(t)$	$f_u(t)\ Ns/m$	$f_w(t)\ Ns/m$	$J_u(t)\ Kg\ m^2$	$J_w(t)\ Kg\ m^2$
0.032	[0 10]	0.098	0.675	0.220	2 .15420	2.15420
0.0485	[10 20]	0.0909	0.675	0.360	2.15419	2.15422
0.065	[20 30]	0.0800	0.675	0.548	2.15418	2.15424
0.0815	[30 40]	0.0631	0.675	0.872	2.15417	2.15425
0.098	[40 50]	0.0320	0.675	2.067	2.15413	2.15426

So, for each line in the above table we can construct an LTI system as follows:

$$\begin{cases} E(t_i)\dot{X} = A(t_i)X + B(t_i)U \\ \quad Y = CX + DU \end{cases} ,i=0,\dots,4 \tag{3.43}$$

and for $i=0$ we get:

$$E(t_0)=\begin{bmatrix} J_u(t_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & v_0 & 0 & L_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_w(t_0) \end{bmatrix} \quad B(t_0)=\begin{bmatrix} -K_u R_u(t_0) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & K_t R_t & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_w R_w(t_0) \end{bmatrix}$$

$$A(t_0)=\begin{bmatrix} -f_u(t_0) & R_u^2(t_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_0 & -v_0 & E_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -R_2^2 & f_2 & R_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_0 & -E_0 & -v_0 & E_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_f^2 & -f_t & R_f^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_0 & -E_0 & -v_0 & E_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -R_4^2 & -f_4 & R_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & v_0 & -E_0 & -v_0 & E_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_w^2(t_0) & f_w(t_0) \end{bmatrix}$$

Equation (3.40) now can be written as [44]

$$\begin{cases} \dot{X} = E(t_i)^{-1}A(t_i)X + E(t_i)^{-1}B(t_i)U \\ Y = CX \end{cases} \tag{3.44}$$

where

$$E(t_i)^{-1}A(t_i)=\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix},\ E(t_i)^{-1}B(t_i)=\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix},\ C=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Applying the same method shown for the first example we can decompose system (3.44) using overlapping technique as follows:

III.2.4. Problem's solution

The overlapping system (3.44) can be expanded to the system:

$$\begin{cases} \dot{X}_e = (E(t_i)^{-1}A(t_i))_e X_e + (E(t_i)^{-1}B(t_i))_e U_e \\ Y_e = C_e X_e \end{cases} \tag{3.45}$$

Using the transformation matrices defined by

$$X_e = TX,\ U_e = S^I U,\ y_e = Gy$$

and

$$T^I T = I_n = I_9\ ,\ \ SS^I = I_m = I_3,\ G^I G = I_p = I_3$$

and T^I, S^I, G^I indicate the pseudo-inverse of the matrices T, S, G.

$$\begin{cases} \begin{bmatrix} \dot{X}_{1e} \\ \hdashline \dot{X}_{2e} \end{bmatrix} = \left[\begin{array}{cc:cc} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ \hdashline A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{array}\right] \begin{bmatrix} X_{1e} \\ \hdashline X_{2e} \end{bmatrix} + \left[\begin{array}{cc:cc} B_{11} & 0 & 0 & 0 \\ 0 & B_{11} & 0 & 0 \\ \hdashline 0 & 0 & B_{11} & 0 \\ 0 & 0 & 0 & B_{11} \end{array}\right] \begin{bmatrix} u_{1e} \\ \hdashline u_{2e} \end{bmatrix} \\ \begin{bmatrix} y_{1e} \\ \hdashline y_{2e} \end{bmatrix} = \left[\begin{array}{cc:cc} C_{11} & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 \\ \hdashline 0 & 0 & C_{22} & 0 \\ 0 & 0 & 0 & C_{33} \end{array}\right] \begin{bmatrix} X_{1e} \\ \hdashline X_{2e} \end{bmatrix} \end{cases} \tag{3.46}$$

with

$$\begin{cases} X_{1e} = \begin{bmatrix} X_1^T & X_2^T \end{bmatrix}^T, \; X_{2e} = \begin{bmatrix} X_2^T & X_3^T \end{bmatrix}^T \\ u_{1e} = \begin{bmatrix} u_1^T & u_2^T \end{bmatrix}^T, \; \mathrm{u}_{2e} = \begin{bmatrix} u_2^T & u_3^T \end{bmatrix}^T \\ y_{1e} = \begin{bmatrix} y_1^T & y_2^T \end{bmatrix}^T, \; \mathrm{y}_{2e} = \begin{bmatrix} y_2^T & y_3^T \end{bmatrix}^T \end{cases} \tag{3.47}$$

❖ **Theorem 1**

The system (3.46) is expansion of the system (3.45) or the system (3.45) is contraction of the system (3.46) if and only if there exist full transformation matrices T, S, G such that

$$\begin{cases} (E(t_0)^{-1} A(t_0))_e T = T(E(t_0)^{-1} A(t_0)) \\ (E(t_0)^{-1} B(t_0))_e = T(E(t_0)^{-1} B(t_0))S \\ C_e = GCT^I \end{cases} \Rightarrow \begin{cases} (E(t_0)^{-1} A(t_0))_e = T(E(t_0)^{-1} A(t_0))T^I + A_s \\ (E(t_0)^{-1} B(t_0))_e = T(E(t_0)^{-1} B(t_0))S + B_s \\ C_e = GCT^I + C_s \end{cases}$$

A_s, B_s, G_s : are complementary matrices satisfying the condition of the following theorem.

❖ **Theorem 2**

The system (3.46) is an expansion of the system (3.44) if

$$\begin{cases} A_s T = 0 \\ B_s = 0 \\ C_s T = 0 \end{cases} \tag{3.48}$$

with

$$A_s = \begin{bmatrix} 0 & .5A_{12} & -.5A_{12} & 0 \\ 0 & .5A_{22} & -.5A_{22} & 0 \\ 0 & -.5A_{22} & .5A_{22} & 0 \\ 0 & -.5A_{32} & .5A_{32} & 0 \end{bmatrix} \tag{3.49}$$

III.2.6. Summary

We have seen in this chapter application of decentralized optimal overlapping decomposition for real systems, the first was six floor building system, for which we have given brief mathematical description of the system and the process being applied. The second was web winding system (applied for paper, metal …etc) with three motors, unwinding motor, tractor and winding motors.

We conclude that designing an algorithm for such type of systems is possible because it satisfies condition for this algorithm (condition of expansion / contraction, condition of contractibility of controllers)

Chapter four

Simulations' results

In this chapter, we present the MATLAB's graphical user interface which is used to interface the algorithm developed in the whole work, then we show the detailed steps for the user to use correctly these two simulated systems and find out successfully the results according to the need of the user. At the end we discuss the results founded for both building and web winding systems

IV.1. Introduction

This chapter concerned about all simulations done in this work, and also all results found from it, with discussion for each interesting graphs. We first show how to simulate GUI of developed algorithms where we start by GUI of building system for both optimal and non-optimal control and then we move to GUI of web winding system for which we simulate the model for also optimal and non-optimal decentralized optimal control.
We didn't give all results of simulation but just results that seams interesting to our study.

IV.2. Flowchart of algorithm

The flowcharts for different steps of overlapping decomposition are shown as follows:

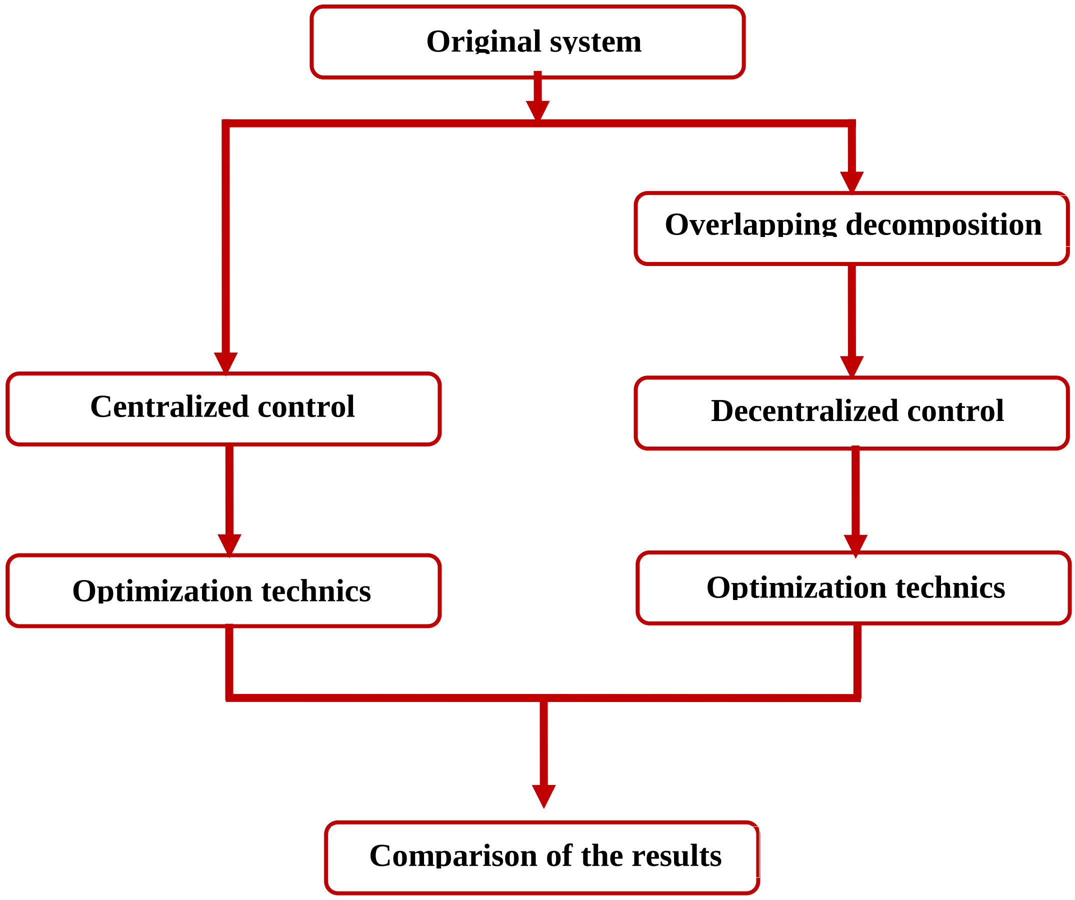


Fig.IV.1. Flow chart of algorithms developed in this work

❖ First, we start by algorithm of decomposition

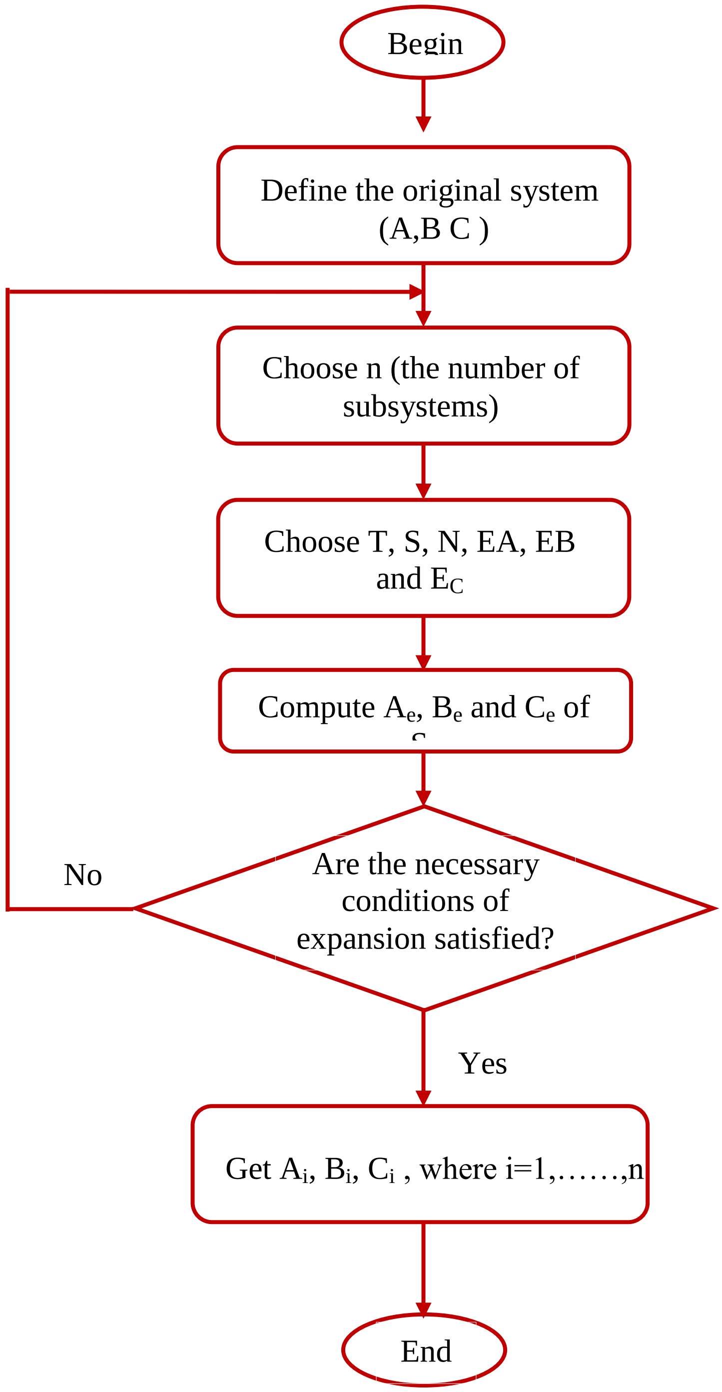


Fig.IV.2. Flow chart for the decomposition of the dynamic system S

❖ Algorithm of optimal decentralized control

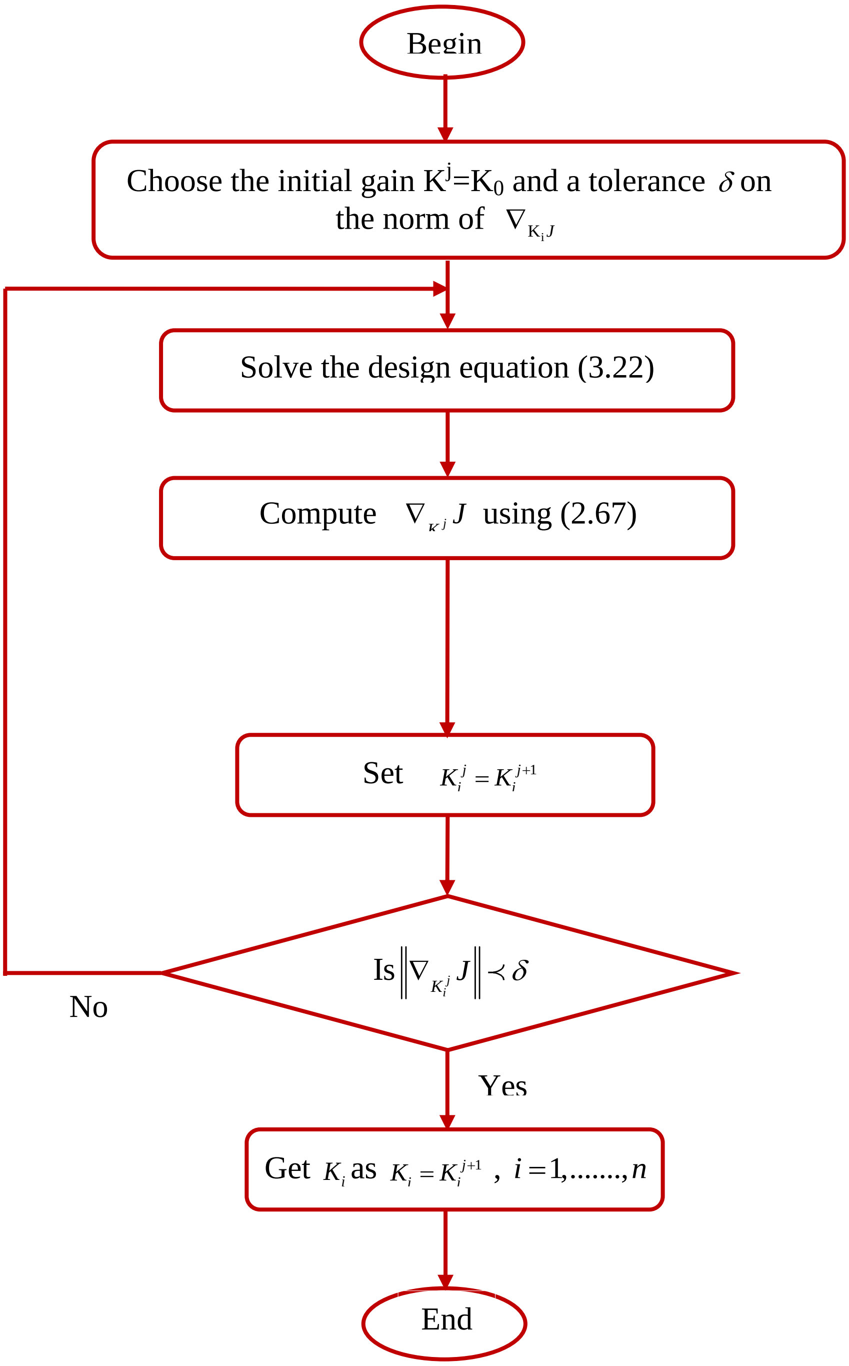


Fig.IV.3. Flow chart for decentralized control of dynamic system

IV.3. Graphical User Interface (GUI)

IV.3.1. What is GUI

Matlab has modest resources to create a Graphical User Interface (GUI), it might be useful for applications where navigation complexity is an issue and to present users with information about a model; the GUI tool included in MATLAB is similar to those found in many high-level languages (drag-drop approach). The idea is to create a simple interface dragging some pre-built object to an empty figure. To start the GUI tool type >>guide at the command prompt.

V.3.2. GUI Development

GUIDE, MATLAB's Graphical User Interface development provides a set of tools for laying out your GUI; the layout editor is the control panel for GUIDE. To start the layout Editor uses the guide command. The following picture shows the layout Editor with the show names in component palette preference selected [45].

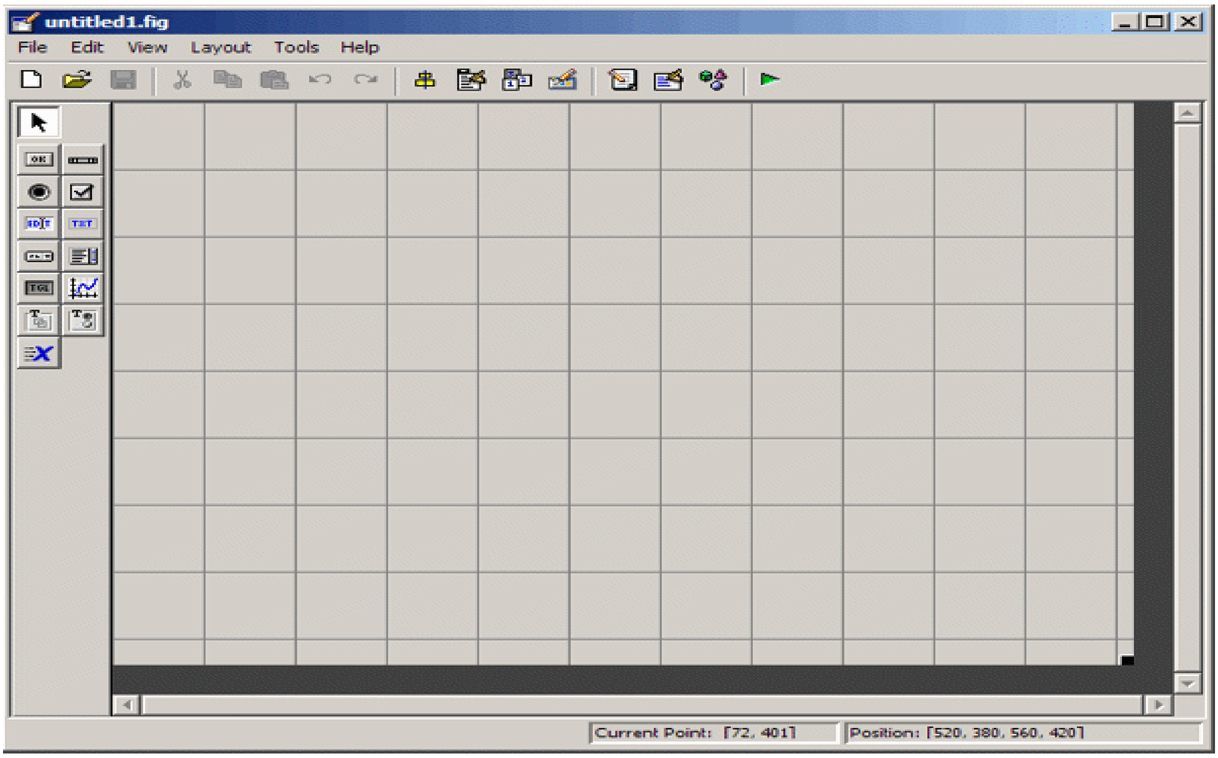


Fig.IV.4. Layout editor of GUI with its entire component

Creating a GUI consists of two steps. Step one: is to design the user interface including figures, buttons and menus. Step two: is to create the MATLAB code that runs when buttons are pressed. The code goes in callback functions that are activated each time a button is pressed.

To make things simple, Matlab includes a GUIDE application that provides a WYSIWYG interface for laying out components on the screen and creates stub callback functions. GUIDE is ideal for most simple GUI applications.

Start GUIDE from the MATLAB command line   >>**guide**

IV.3.3. How to create a GUI

In the MATLAB command window: >>**guide**: opens GUIDE quick start then select blank GUI.

File ->Preferences ->GUIDE: Show names component palette.

View ->Property inspector: Click and drag to resize girded layout area to desired size, or use position property in Property Inspector to set an exact size.

Tools -> Application Options ->

- ✓ Resize behavior = Proportional.
- ✓ Add Components.
- ✓ Use Static Text component for labels.
- ✓ Align components using the Align tool bar button.
- ✓ Label component using string property.
- ✓ Label figures using Name property.
- ✓ For Popup Menu, String property gets one line per menu entry.
- ✓ Tag components, Tag property; e.g. button ‘’go”.

Save GUI using tools: Run will save both a Figure file and M file under the chosen name (“untitled2010 “in this project).

IV.3.4. How to create a GUI’s code

Place code that should run when the application starts in the opening function which is called guiname –openingFcn (). Place code after comments (begins with %). Place codes for buttons and other components in the appropriate callback function stubs that are at the end of the M file [45].

IV.3.5. GUI applications

The GUI is used in this study to represent the two real application of the proposed method. We have used in this interface mathematical models for two real systems; the first which is building system contains six degree of freedom and the second is web winding system with three motors. To simplify the use of our GUI we have developed directive GUI for selecting example wanted to be simulated.

You can use this interface to select system for simulation as follows:

1. Open “untitled2010” GUI from MATLAB GUI files

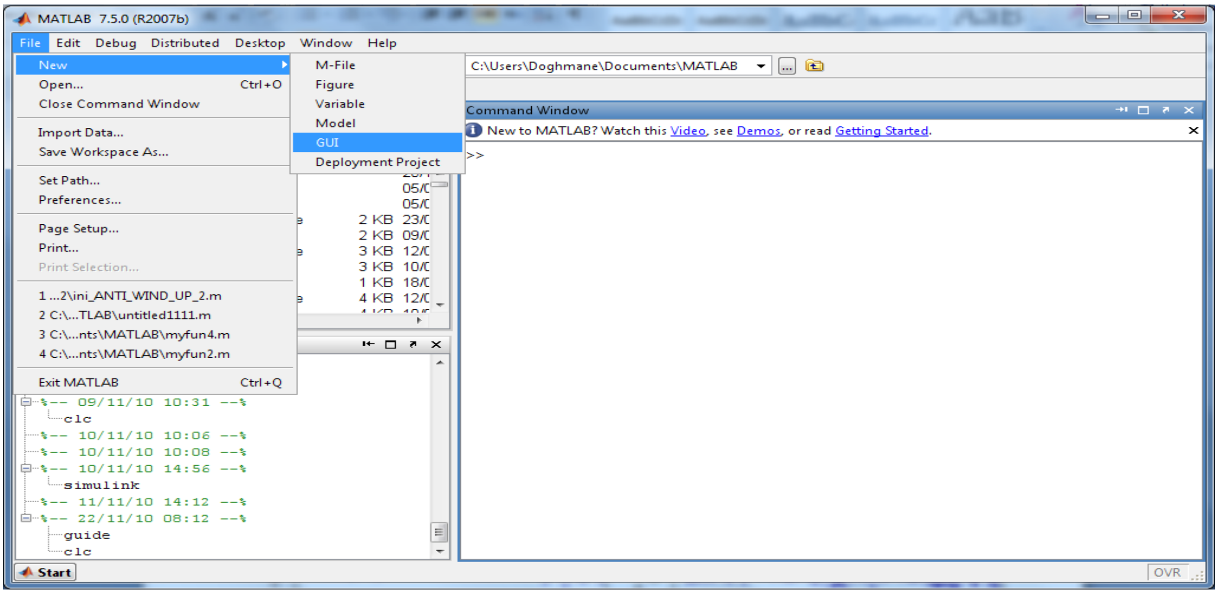


Fig.IV.5. First step of opening a Matlab GUI

The select “c:\Users\Doghmane\Documents\Matlab\Untitled2010.fig” from “open Existing GUI” as shown below:

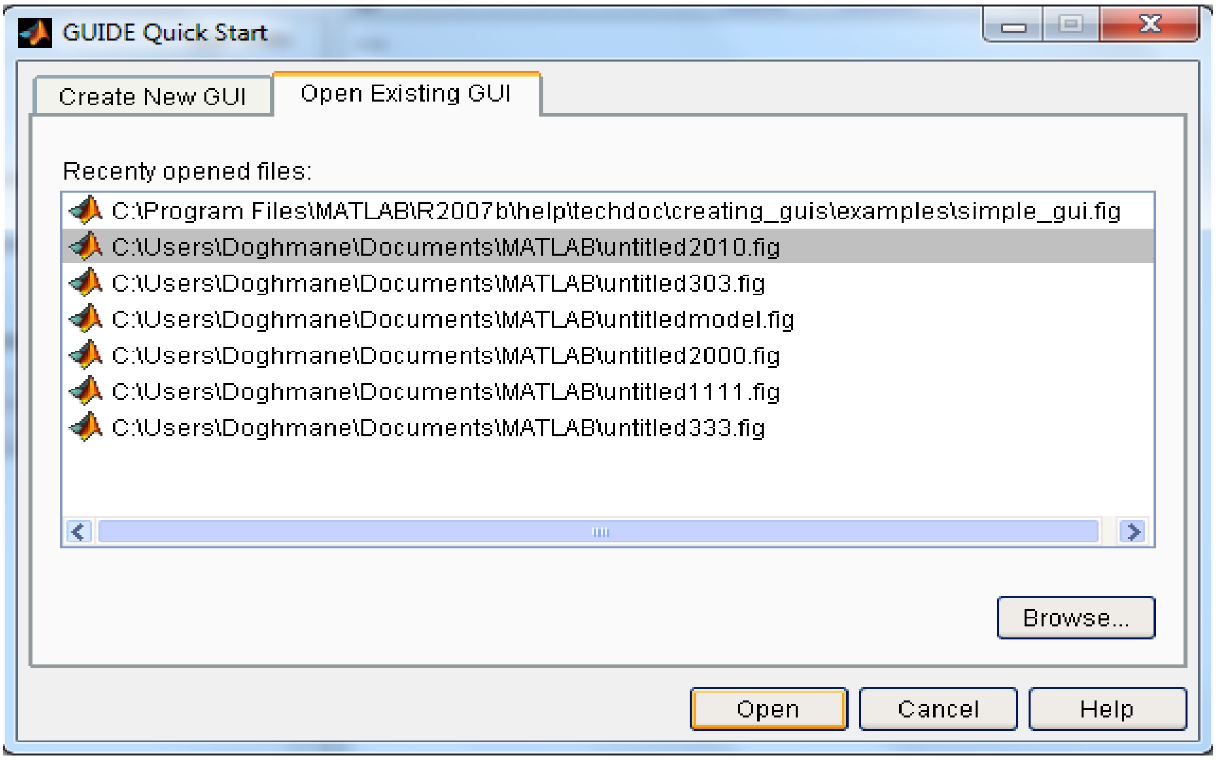


Fig.IV.6. Second step of opening a Matlab GUI

Then, select system that you choose to simulate

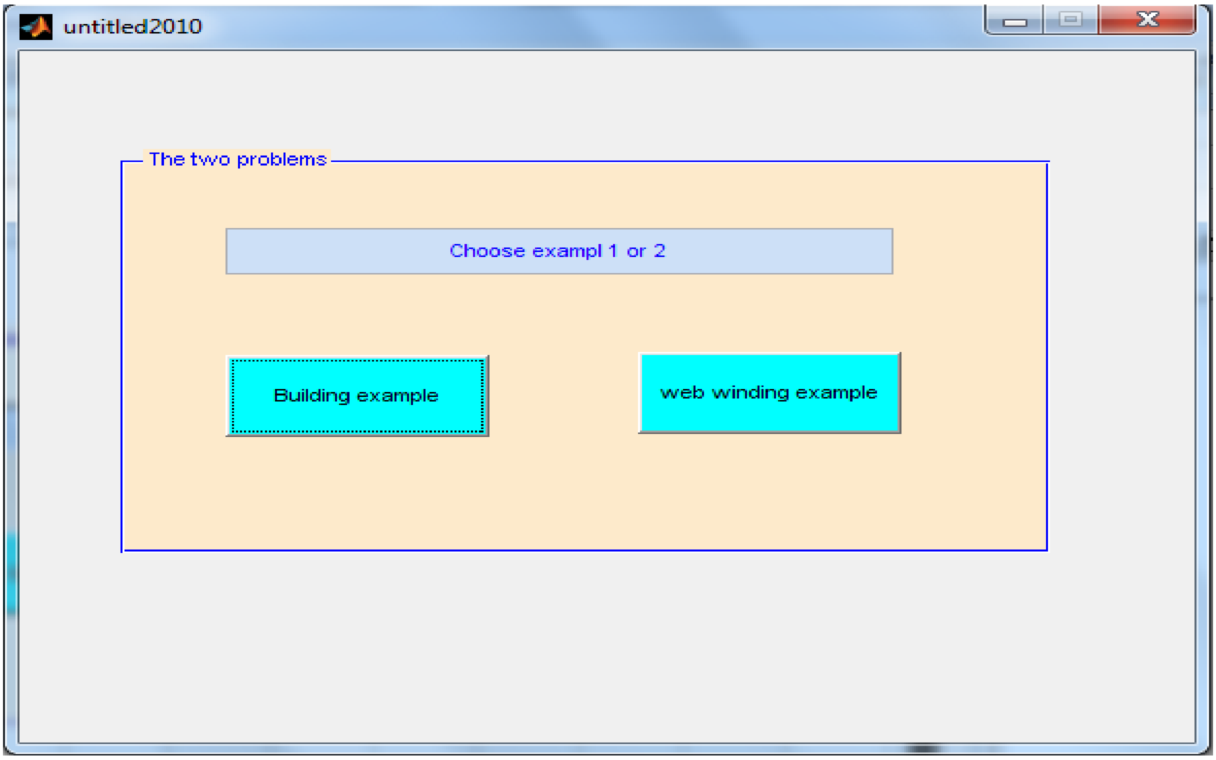


Fig.IV.7. Selecting step of the chosen system.

IV.3.6. GUI of building system

From the selective GUI we choose “building example”, this will guide us to the GUI of building system

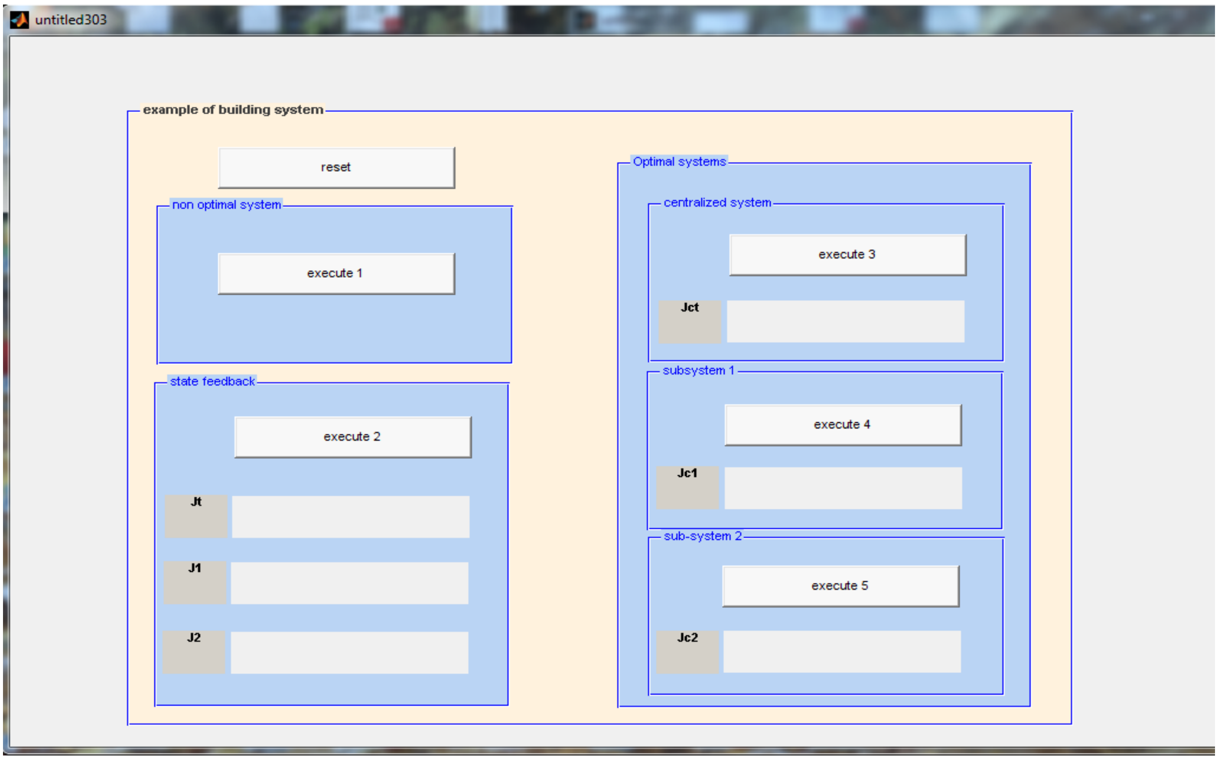


Fig.IV.8. GUI of building system with optimal and non-optimal simulation.

V.3.7. Results for building system

Results founded in this paper using Matlab 2007 simulation, where first, we started with centralized non-optimal controller for each floor as shown in figures below. First; we start by “execute1” which allows us to simulate non-optimal system where here we care only of minimizing the response of the building to earth quick disturbances

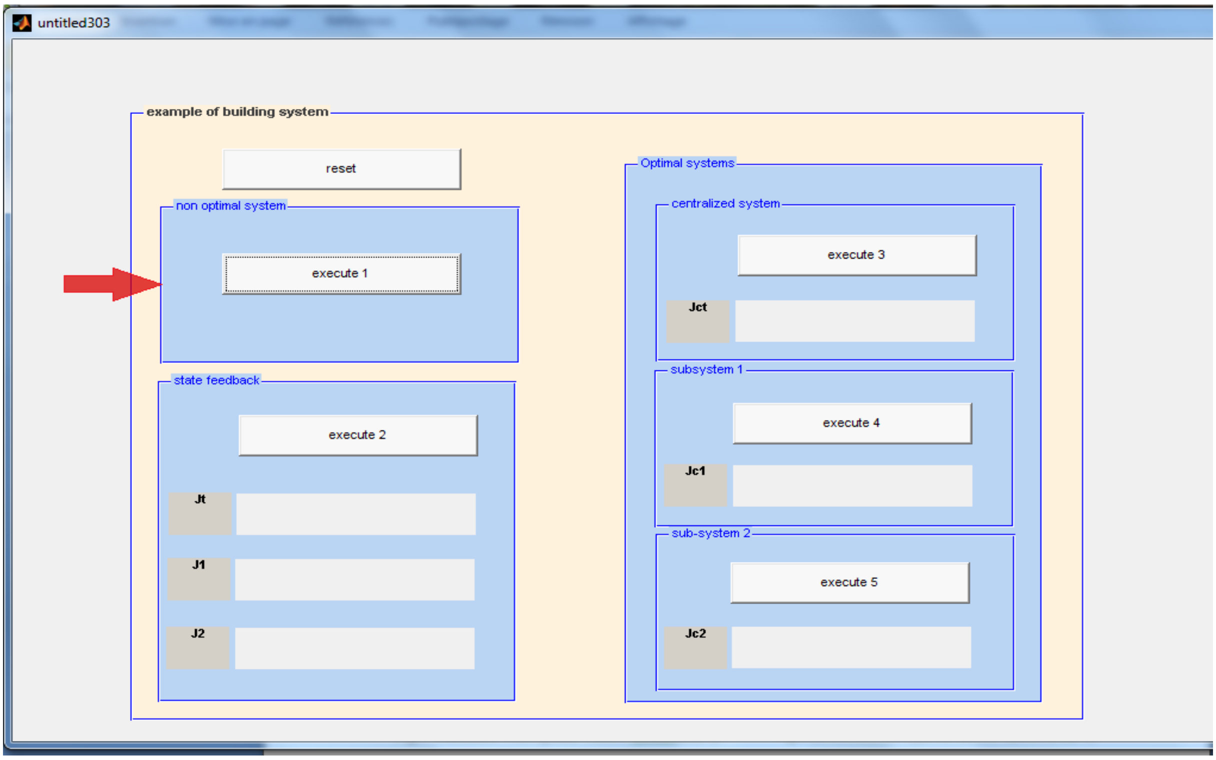


Fig.IV.9. Simulation of non-optimal control of building system.

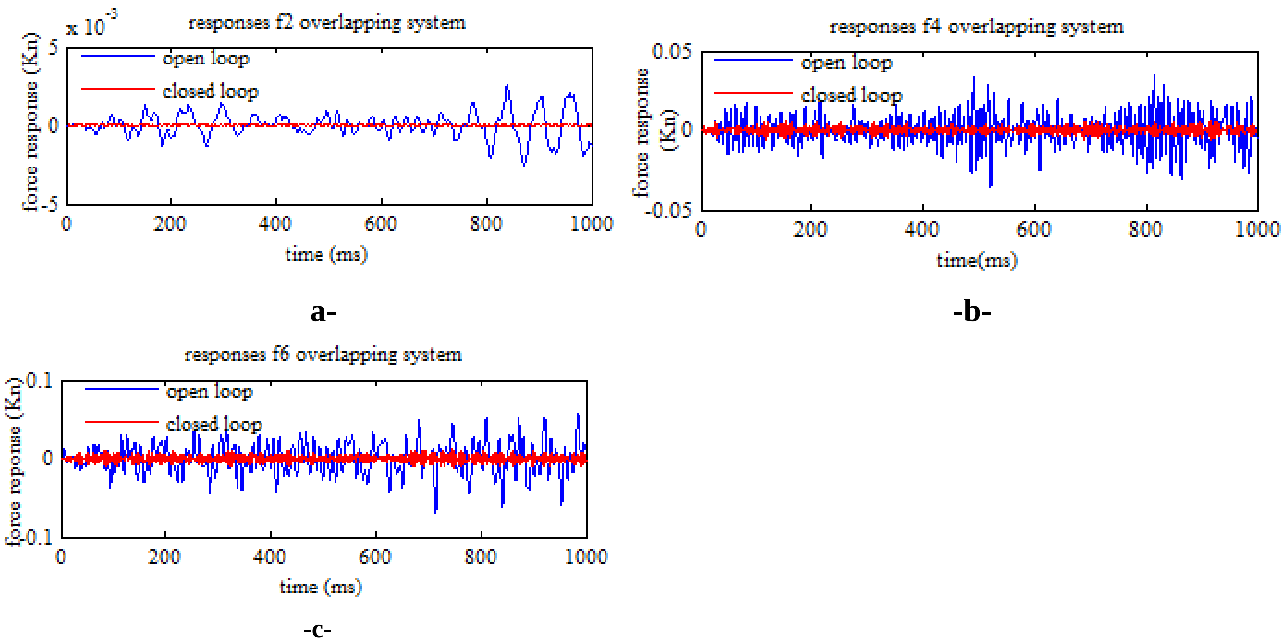
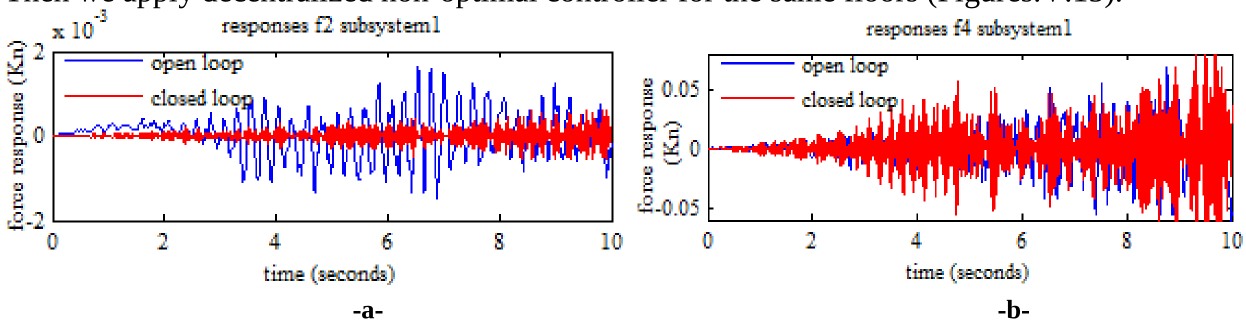
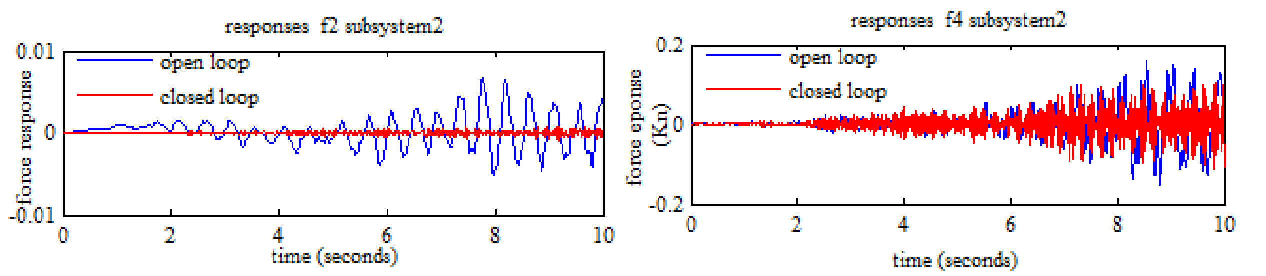


Fig.IV.10. Non-Optimal centralized control system; a- Flour2, b-Flour 4, c-Flour 6

Then we apply decentralized non-optimal controller for the same floors (Figures.V.15).



-a- -b-



-c- **-d-**

Fig.IV.11. Non-Optimal decentralized control; a- subsystem 1 flour2, b- subsystem 1 flour 4, c- subsystem 2 Flour2, d- Subsystem 2 Flour 4

We notice that in common floor (Fig.IV.11,b and d) the response of system in closed loop form still effective ; this is according to the interconnection between subsystems.
An optimization technique was applied for centralized and decentralized controller as show in figure below:

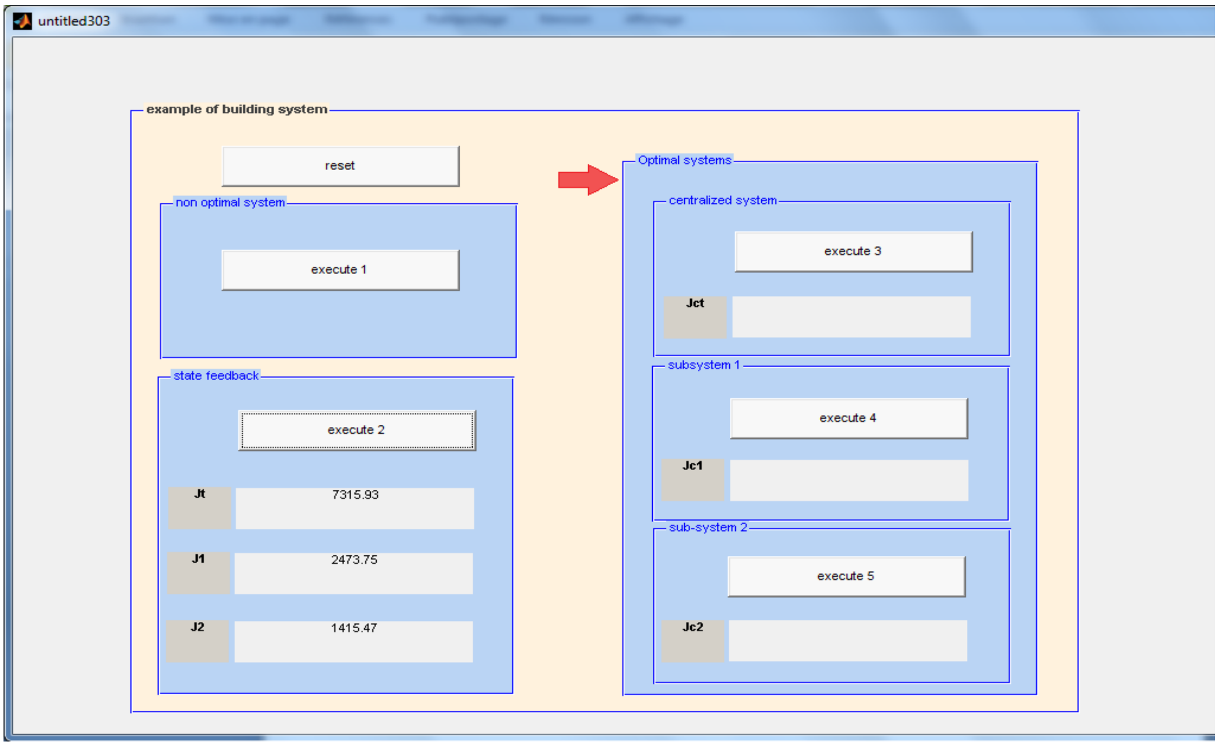
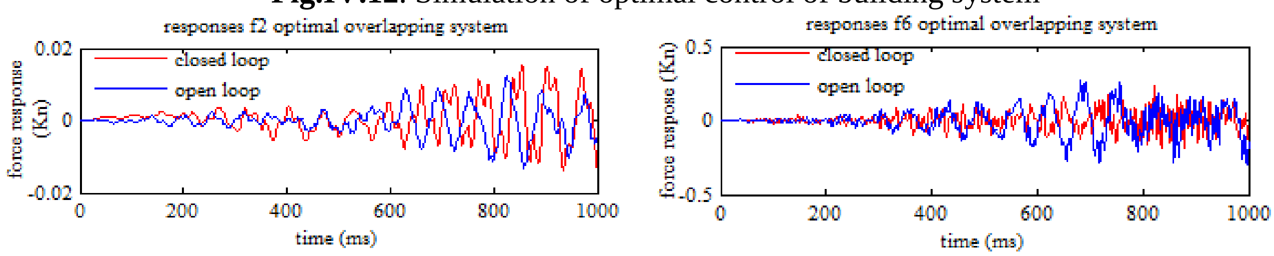


Fig.IV.12. Simulation of optimal control of building system



-a- -b-

Fig.IV.13. Optimal centralized control system; a- Flour 2, b- Flour 6

Even when we applied optimization responses in Fig.12 still valuable which may make damages to the building. The cost function of centralized controller is $J_t = 7.31*10^3$ while decentralized controller the

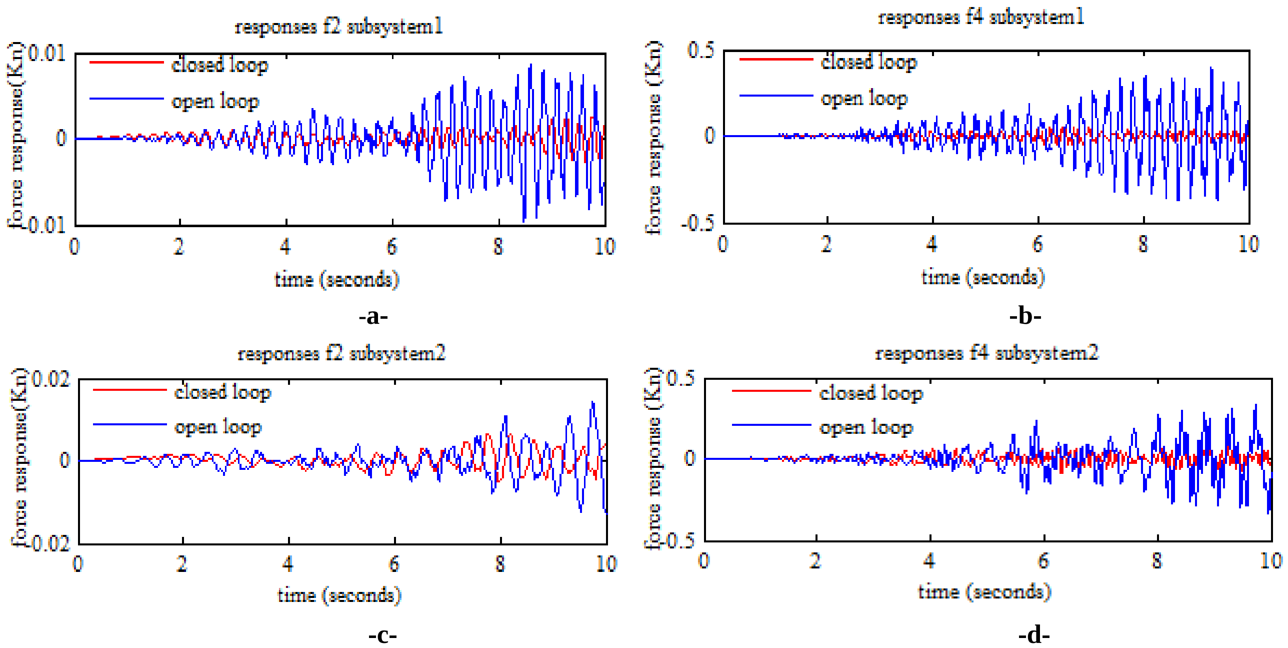


Fig.IV.14. Optimal decentralized control ; a- Subsystem1 Flour 2,b- Subsystem 1 Flour 4, c- Subsystem2 Flour 2, d- Subsystem 2 Flour 4

cost functions of subsystems 1 and 2 are respectively $J_1 = 2.47*10^3$ and $J_2 = 1.41*10^3$.

The overlapping decomposition method has been presented in this example, and then the inclusion principle has been introduced to provide a mathematical framework for decentralized control. The inclusion of the cost function has also been discussed to incorporate the optimal control problem for large scale systems where the concept of contractibility of controllers has been discussed. Optimal decentralized dynamic output feedback controllers design has been proposed for building systems with overlapping structure. Non-optimal overlapping centralized and decentralized controllers are designed for which we found that decentralized controller give better results. Furthermore to improve these results we developed an optimization technique that allow us not just design optimal controller but also minimize the cost function of the whole system for decomposed decentralized ($J_1 = 2.47*10^3$, $J_2 = 1.41*10^3$) in comparing to centralized ($J_t = 7.31*10^3$) controller; where we it is clear that $J_1 + J_2 \prec J_t$.

IV.3.8. GUI of web winding system

From the selective GUI, we choose “web winding example”, this will guide us to the GUI of web winding system as shown in Figure bellow

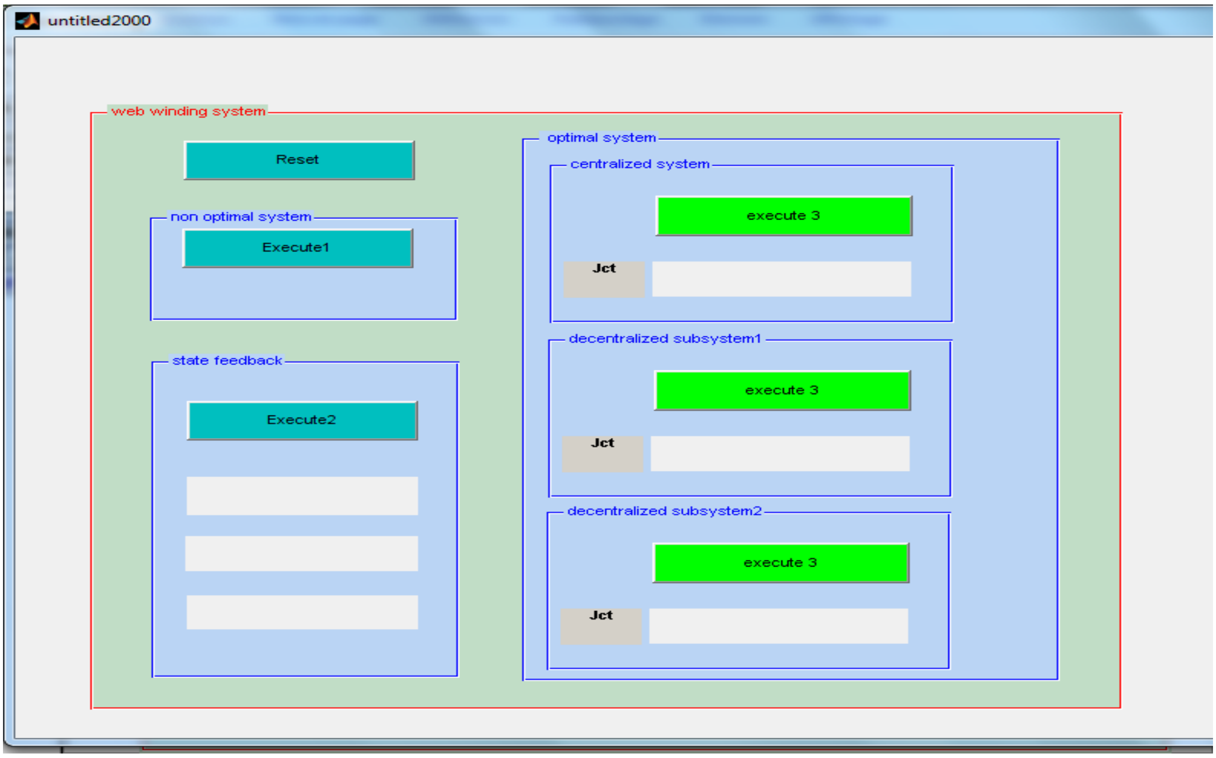
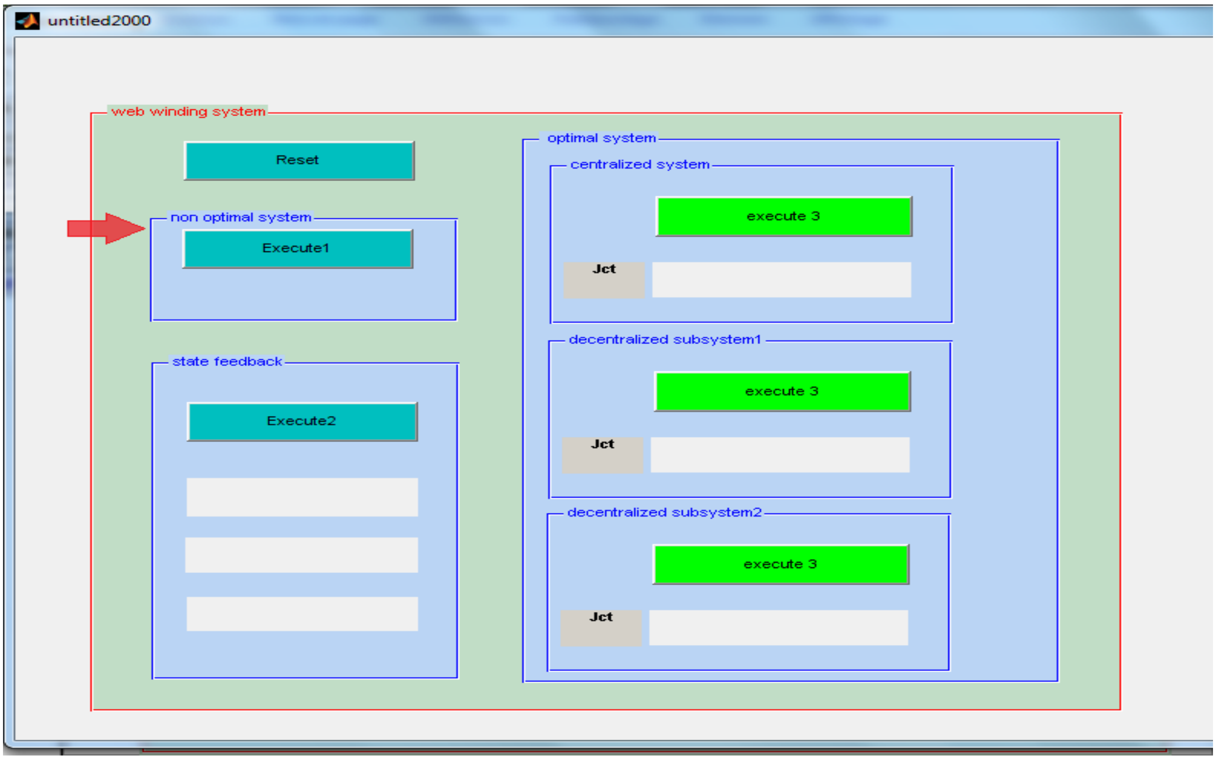


Fig.IV.15. GUI of web winding system with optimal and non-optimal simulation.

IV.3.9. Results for web winding system

Results founded in this paper using Matlab 2007 simulation, where first, we started with centralized non-optimal controller for each motor.

First; we start by “execute1” which allows us to simulate non-optimal system where here we care only of stabilizing the response of the web winding system to disturbances.

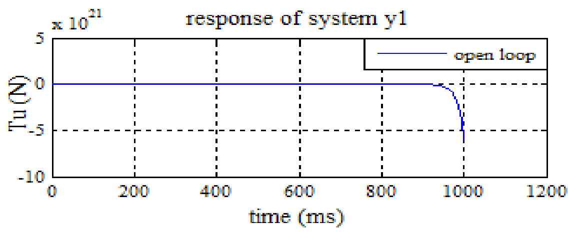


FigV.16. Simulation of non-optimal control of web winding system.

1. Non-optimal centralized control

In this part, our goal is to stabilize the response of each controller where y_1 is the response of unwinder motor (tension T_u), y_2 is the response of tractor motor (velocity V_3) and y_3 is the response of winder motor (tension T_w). To simulate this system as an LTI system, we consider a range times t_0, t_1, t_2, t_3 and t_4 where for each range time we found

- For t_0



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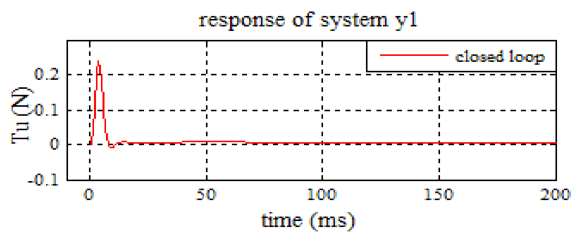

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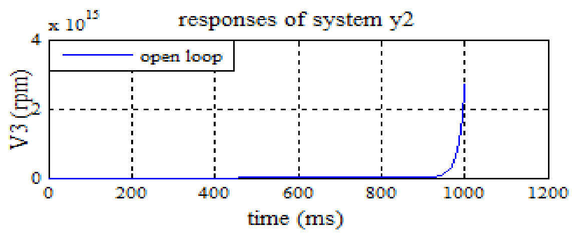

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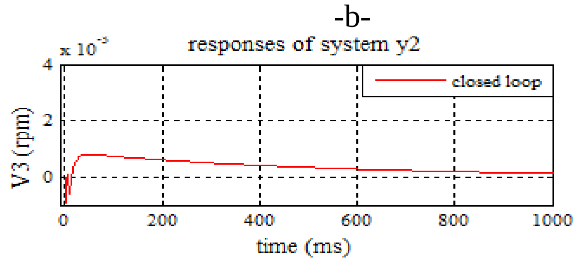

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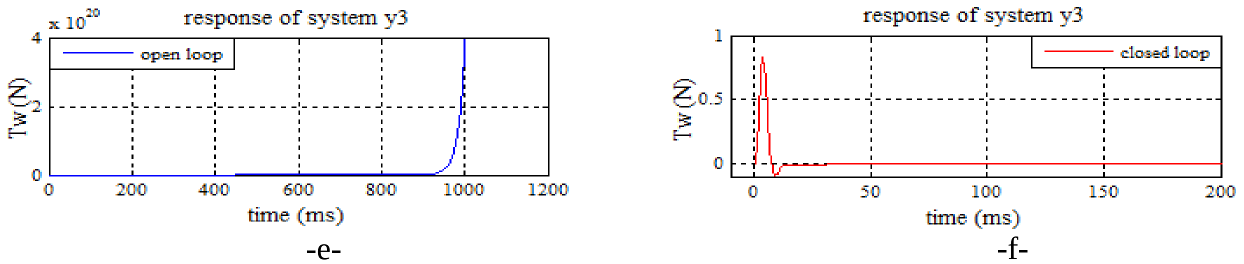


Fig.IV.17. Non-optimal centralized responses; a,b- unwinder motor , c,d- Second motor, e,f- winder motor.

The left sides of Fig.V.17 represents the open loop responses for unwinder, tractor and winder motors respectively, however the right sides are the closed loop responses of the same motors where

***For unwinder motors:** the open loop response (tension value) goes to negative infinity ($-5*10^{21}$) when time response is close to 10s, however for centralized closed loop response , it has a peak of $.23$ at $t=50ms$ then it back to zero for less than $t=0.2s$.

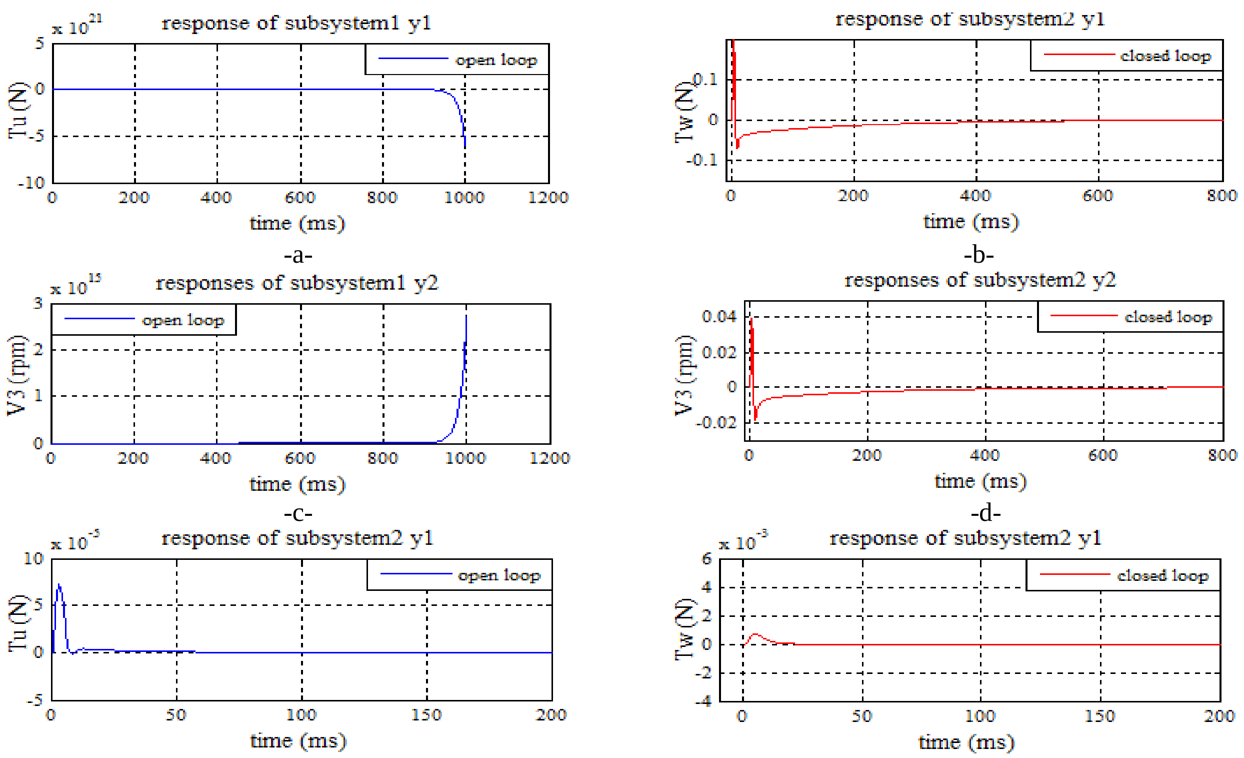
***For tractor motor (common part):** also the open loop response goes to positive infinity ($2.5*10^{15}$) when $t\to 10s$ however, for centralized closed loop response it has long response of about 10s; this may be according to the interaction between unwinder and winder subsystems in this region.

***For winding motor:** the open loop response (tension value) goes to $4*10^{20}$ for $t\to 10s$, however it will be stabilized for less than $t=0.2s$ after a peak of 0.8 at $t=40ms$

As we see here, the goal of stabilizing unwinder and winder and tractor response is reached with centralized control but the response of overlapped area have a long response this is according to the interaction between subsystem of unwinder motor and subsystem of winding motor.

Such long response is not acceptable in these industrial systems because time is most important factor in transporting industry in addition to the damage to the transported material.

To solve such problem we have applied overlapping decomposition that allows us have to separate systems with separate decentralized controller as described before where, the first subsystem contains the unwinding motor y_1 and decomposed tractor motor response y_2 ;subsystem1(y_1, y_2) and subsystem contains the winding motor response and decomposed tractor motor response subsystem2(y_1, y_2) as it is shown in figure below:



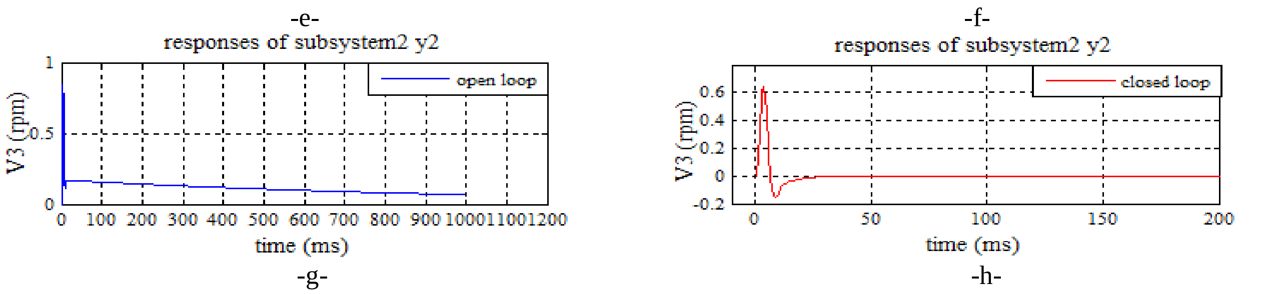


Fig.IV.18. Non-optimal decentralized responses for t_0; a, b, c, d: subsystem 1, e, f, g, h: subsystem 2

Here, we notice overlapping decomposition allows us to design decentralized controller for each subsystem, where the response of overlapped are in original system has long response (10s), however here it is only 2s (see Fig.IV.18. f) for subsystem 1 and .2s (see Fig.IV.18.h) for subsystem 2. For the next range time (t_1, t_2, t_3 and t_4), we will show results of decentralized control since it is our interest in the next parts.

- For t_1

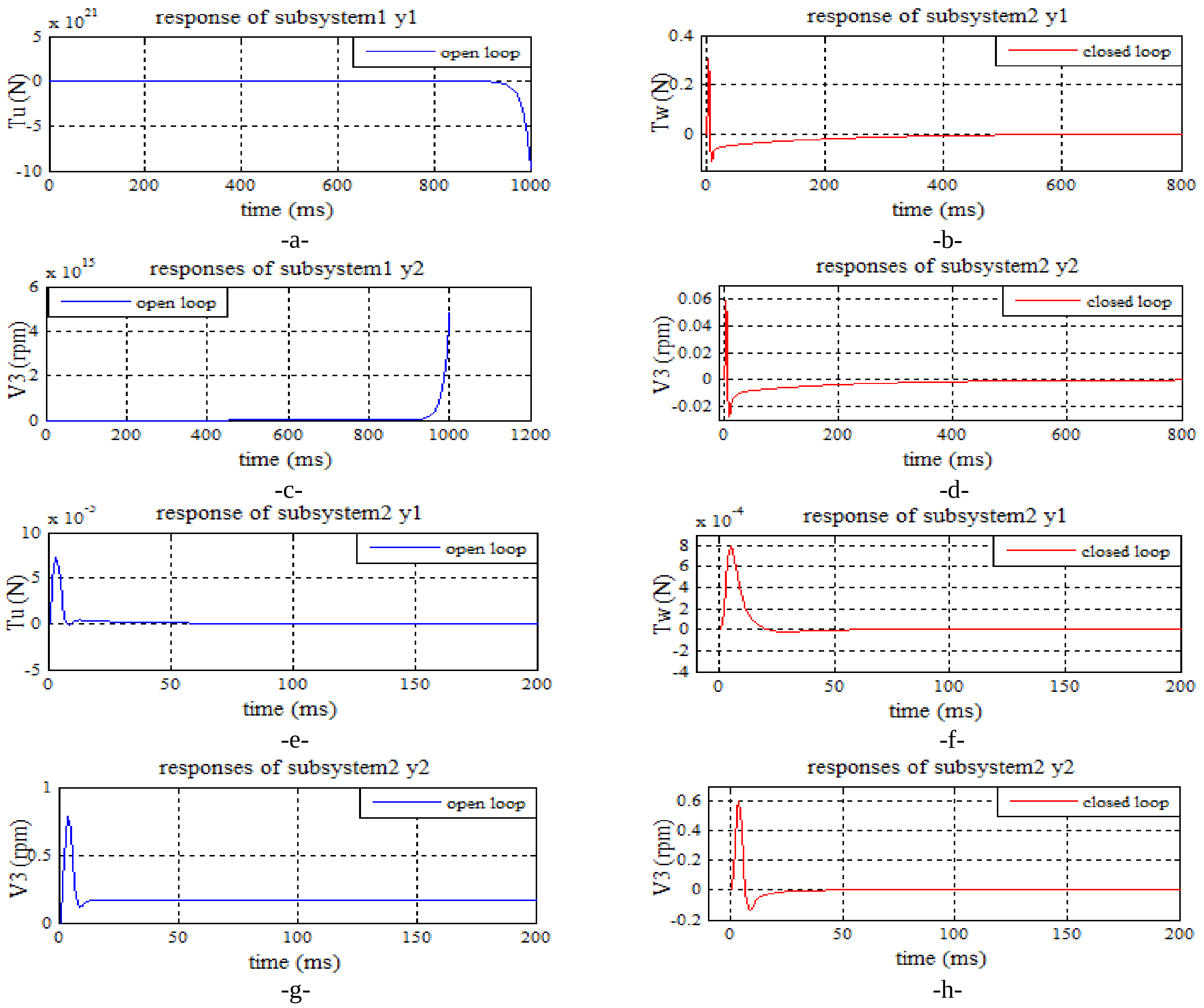


Fig.IV.19. Non-optimal decentralized responses for t_1; a, b, c, d: subsystem 1, e, f, g, h: subsystem 2

***For subsystem1in Fig.IV.19**: unwinding motor is stabilized for $t = 3s$ with peak of 0.3 however the decomposed tractor motor response is stabilized at $t = 3s$ but peak of 0.05.

***For subsystem2 in Fig.IV.19**: winding motor response is stabilized in 0.2s with peak of $0.7*10^{-4}$ at .05s, however the decomposed tractor motor response is stabilized in 0.2s too but with peak of 0.6.

- For t_2

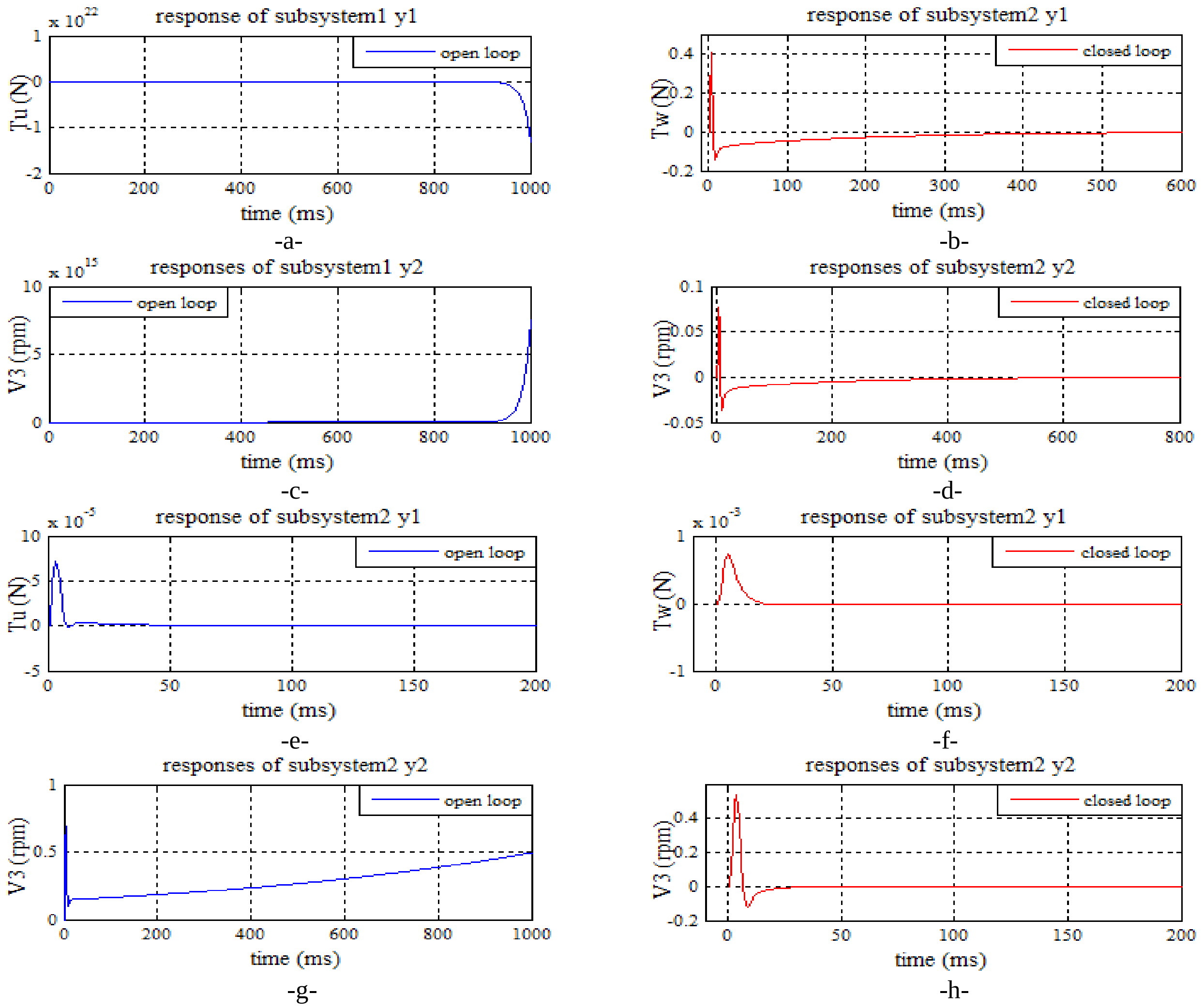
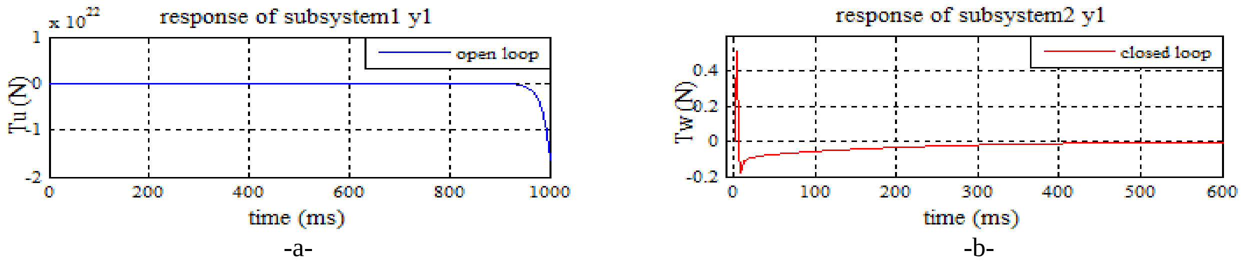


Fig.IV.20. Non-optimal decentralized responses for t_2 ; a, b, c, d: subsystem 1, e, f, g, h: subsystem 2

*__For subsystem1in Fig.IV.20__: unwinding motor is stabilized for $t = 3s$ as for t_1 but with peak of 0.4 however the decomposed tractor motor response is stabilized at $t = 3s$ too with same peak of t_1 .

*__For subsystem2 in Fig.IV.20__: winding motor response is stabilized in less than 0.2s with neglected peak, however the decomposed tractor motor response is stabilized in 0.2s too with same peak as in t_1 .

- For t_3



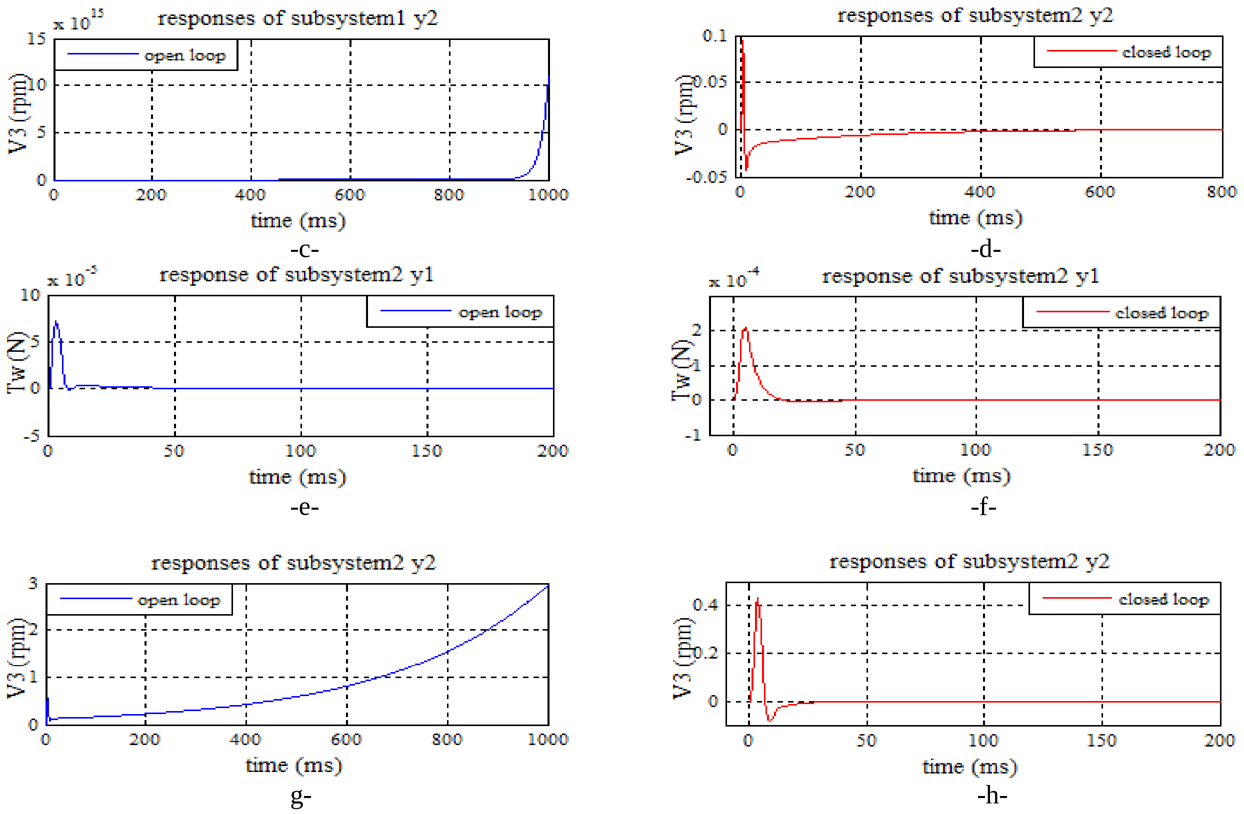
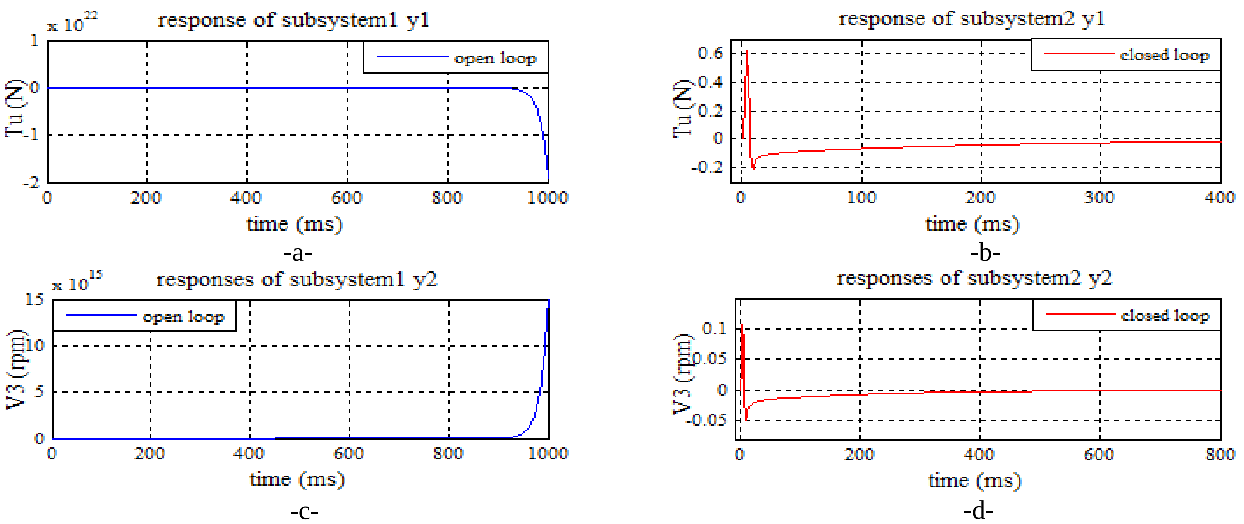


Fig.IV.21. Non-optimal decentralized responses for t_3; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2

*__For subsystem1 in Fig.IV.21__: unwinding motor is stabilized for $t = 2.5s$ with peak of 0.5 however the decomposed tractor motor response is stabilized at $t = 3s$ but peak of 0.1

*__For subsystem2 in Fig.IV.21__: winding motor response is stabilized in 0.2s with peak of 0.8 at .05s, however the decomposed tractor motor response is stabilized in 0.08s with peak of $8*10^{-5}$.

- For t_4



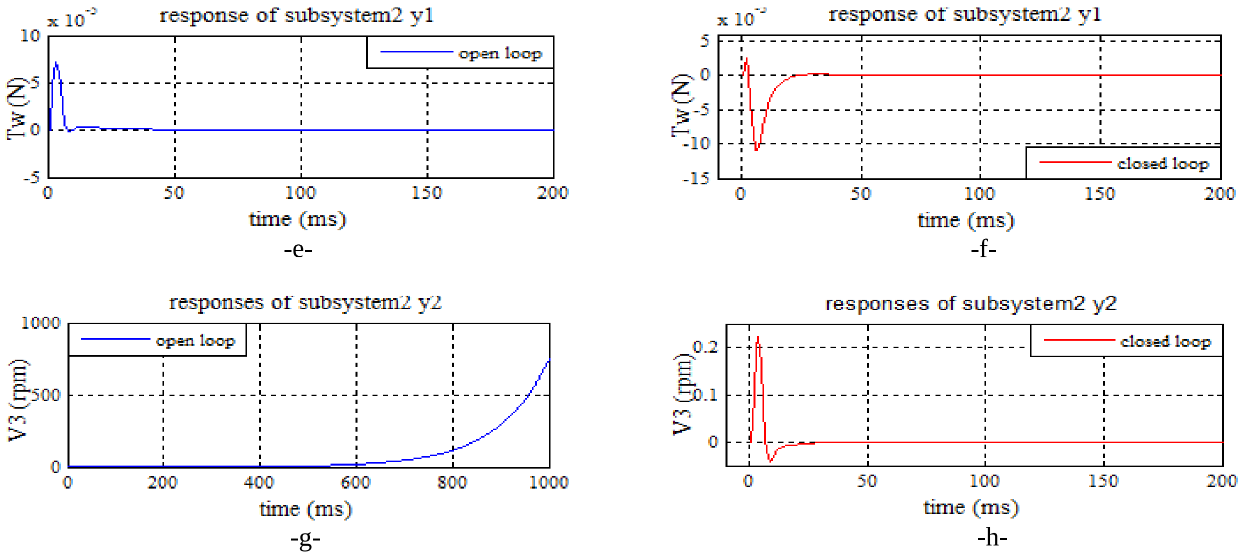


Fig.IV.22. Non-optimal decentralized responses for t_4; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2

***For subsystem1 in Fig.IV.22**: unwinding motor is stabilized for $t = 3s$ with peak of 0.6 however the decomposed tractor motor response is stabilized at $t = 3s$ and peak of 0.1 as in t_3

***For subsystem2 in Fig.IV.22**: winding motor response is stabilized in 0.2s with negative peak of 10^{-4} at .07s, however the decomposed tractor motor response is stabilized in 0.02s with peak of 0.21.

It is noticeable that response of unwinding, tractor and winding motors have the same shape response separately for different regions values of time (t_0, t_1, t_2, t_3 and t_4) , it differs only in amplitude and time needed for stabilizing each motor of the subsystems.

Till now, we have successes in designing decentralized controller using overlapping decomposition in order to stabilize the decomposed systems locally then globally which means we have solved the control problem, but in practice the economic side of such industrial systems drive us the solution of the control problem in terms of minimizing a cost function and maximizing the benefits of the controlled system. Optimal control theory is concerned with obtaining controller with minimum cost function (desired value) and at the same time stabilize the system with certain performance measures.

For optimal decentralized system we found the following results:

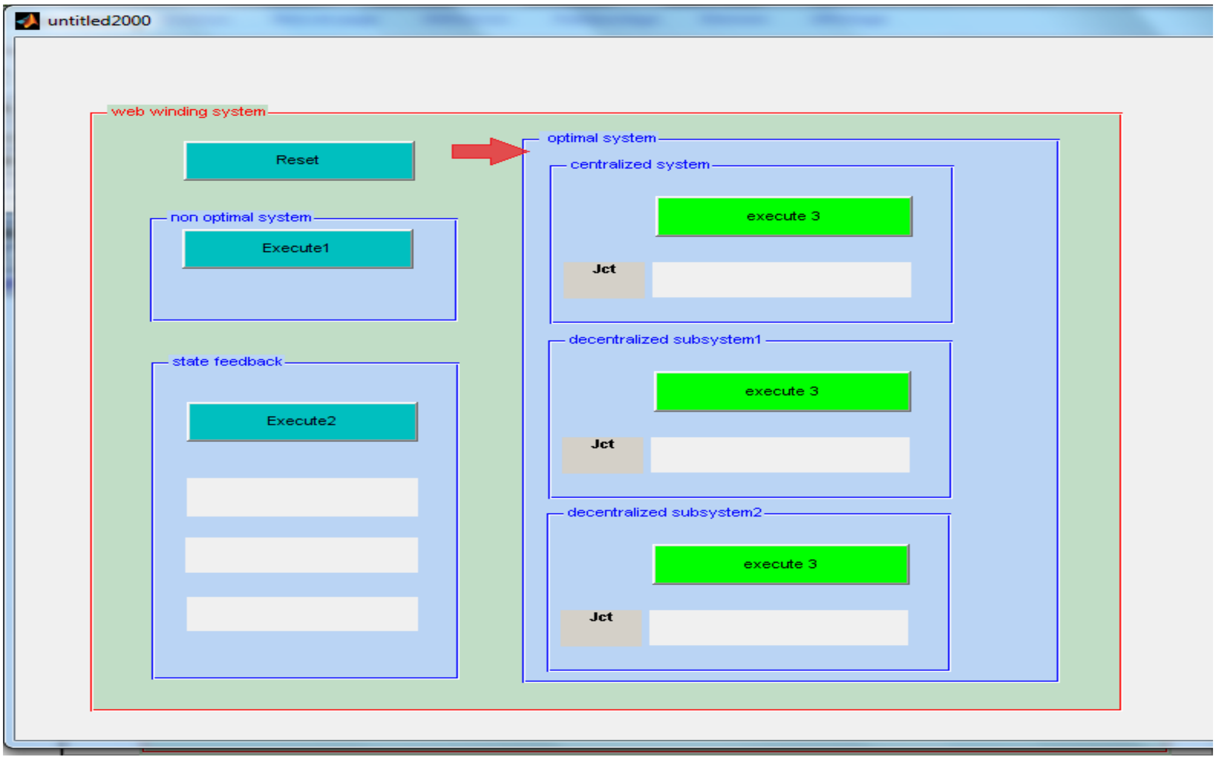
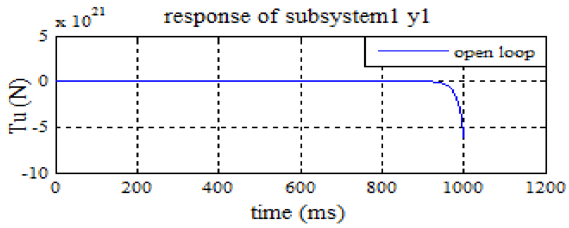


Fig.IV.23. Simulation of optimal control of web winding system.

- For t_0



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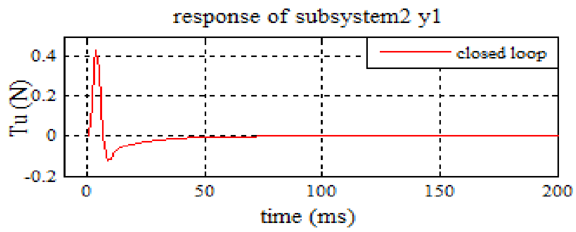

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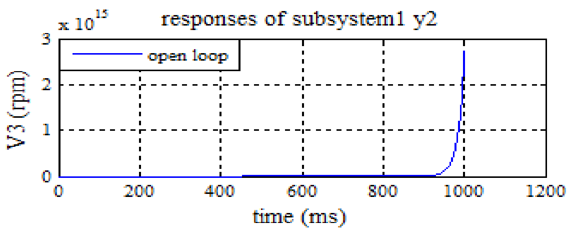

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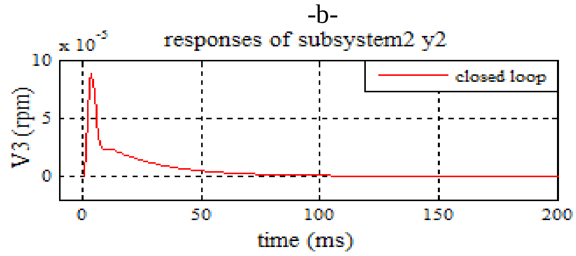

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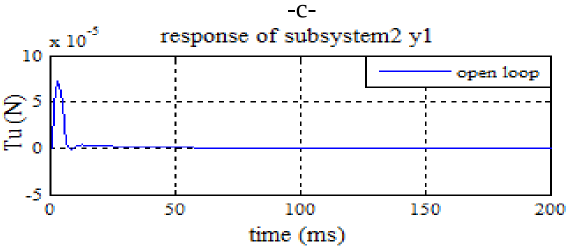

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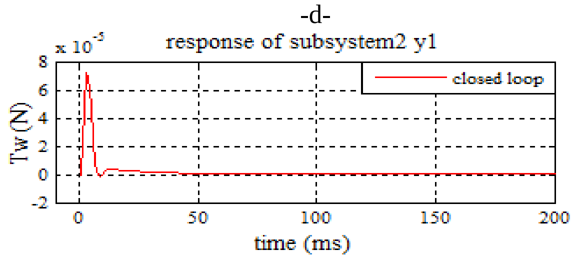

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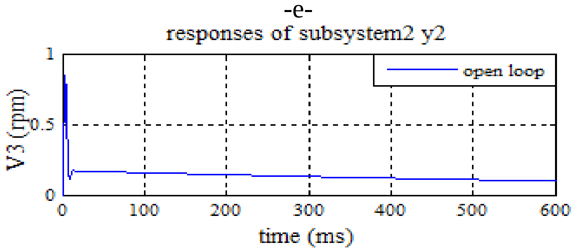

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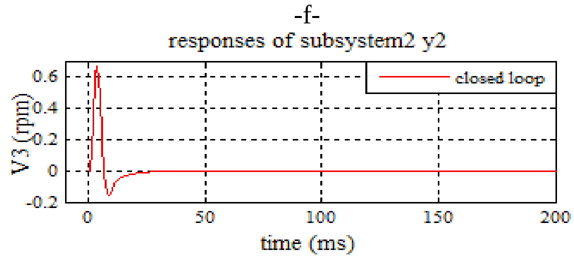

-h-

Fig.IV.24. Optimal decentralized responses for t_0; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2

***For subsystem1in Fig.IV.24**: unwinding motor is stabilized for $t = 6s$ with peak of 0.2 however the decomposed tractor motor response is stabilized at $t = 4s$ but peak of 0.04. The cost function is $J_1 = 1.355 * 10^{14}$

We notice here that stabilization time is larger than in non-optimal algorithm; this is according to optimization algorithm which takes time to calculate the minimum cost function.

***For subsystem2 in Fig.IV.24**: winding motor response is stabilized in 0.2s with peak of $0.7 * 10^{-5}$ at .05s, however the decomposed tractor motor response is stabilized in 0.2s too but with peak of 0.6 as in non-optimal system, the cost function is $J_2 = 431.845$.

In comparing with optimal centralized control which gives cost function of $J_t = 1.439 * 10^{14}$, it is clear that $J_1 + J_2 \prec J_t$; which means that overlapping decomposition doesn't just minimize complexity of calculation but also minimize the cost function of the optimal system.

- ❖ For t_1

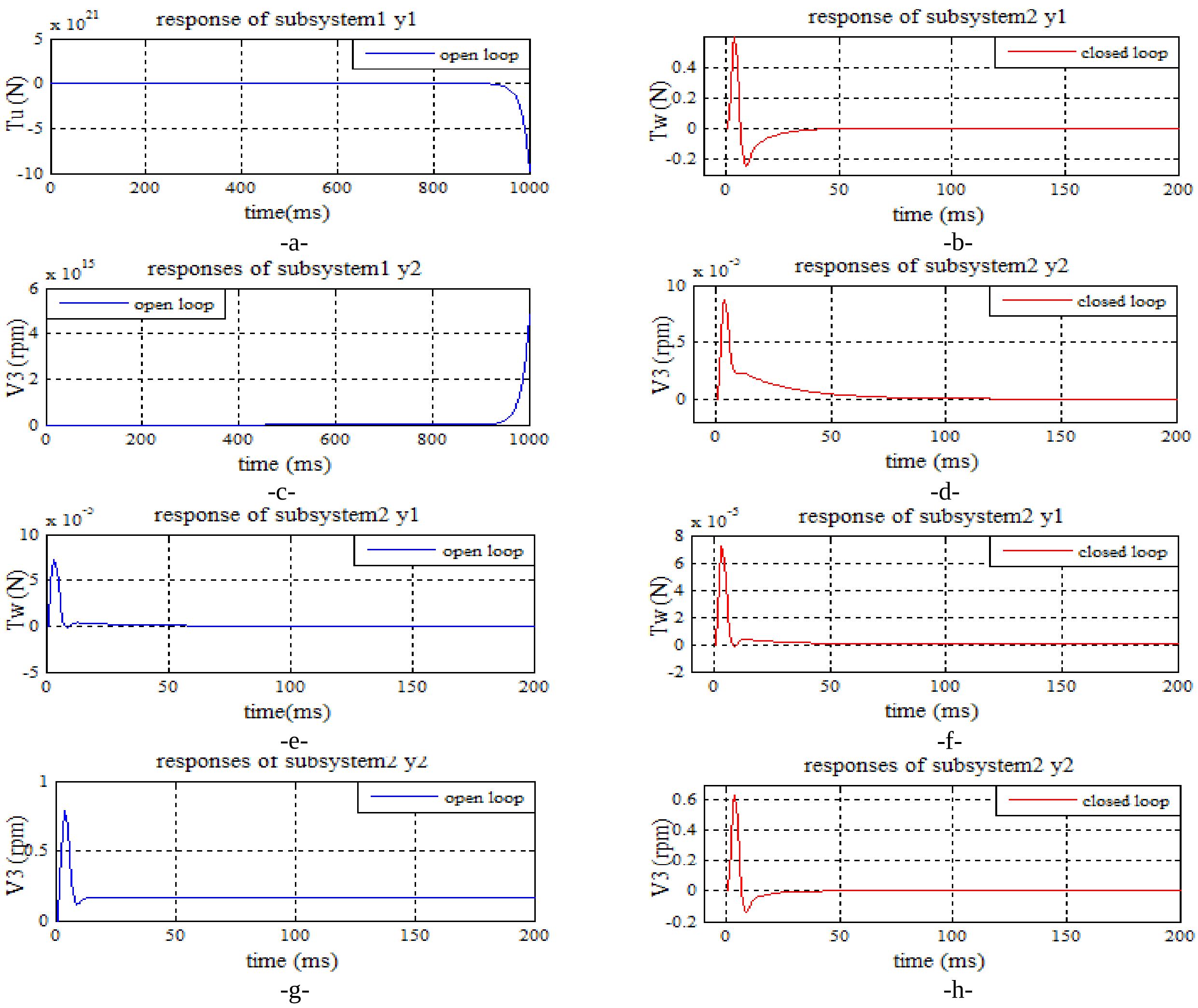

Fig.IV.25. Optimal decentralized responses for t_1; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2

***For subsystem1in Fig.IV.25**: unwinding motor is stabilized for $t = .4s$ with peak of 0.5 however the decomposed tractor motor response is stabilized at $t = .6s$ but peak of $9 * 10^{-5}$. The cost function is $J_1 = 1.335 * 10^{14}$.

***For subsystem2 in Fig.IV.25**: winding motor response is stabilized in 0.2s with peak of $0.7 * 10^{-5}$ at .05s, however the decomposed tractor motor response is stabilized in 0.2s too but with peak of 0.6 as in non-optimal system, the cost function is $J_2 = 431.845$.

In comparing with optimal centralized control which gives cost function of $J_t = 1.4398 * 10^{14}$, it is clearly that $J_1 + J_2 \prec J_t$; which means that overlapping decomposition doesn't just minimize complexity of calculation but also minimize the cost function of the optimal system.

❖ For t_2

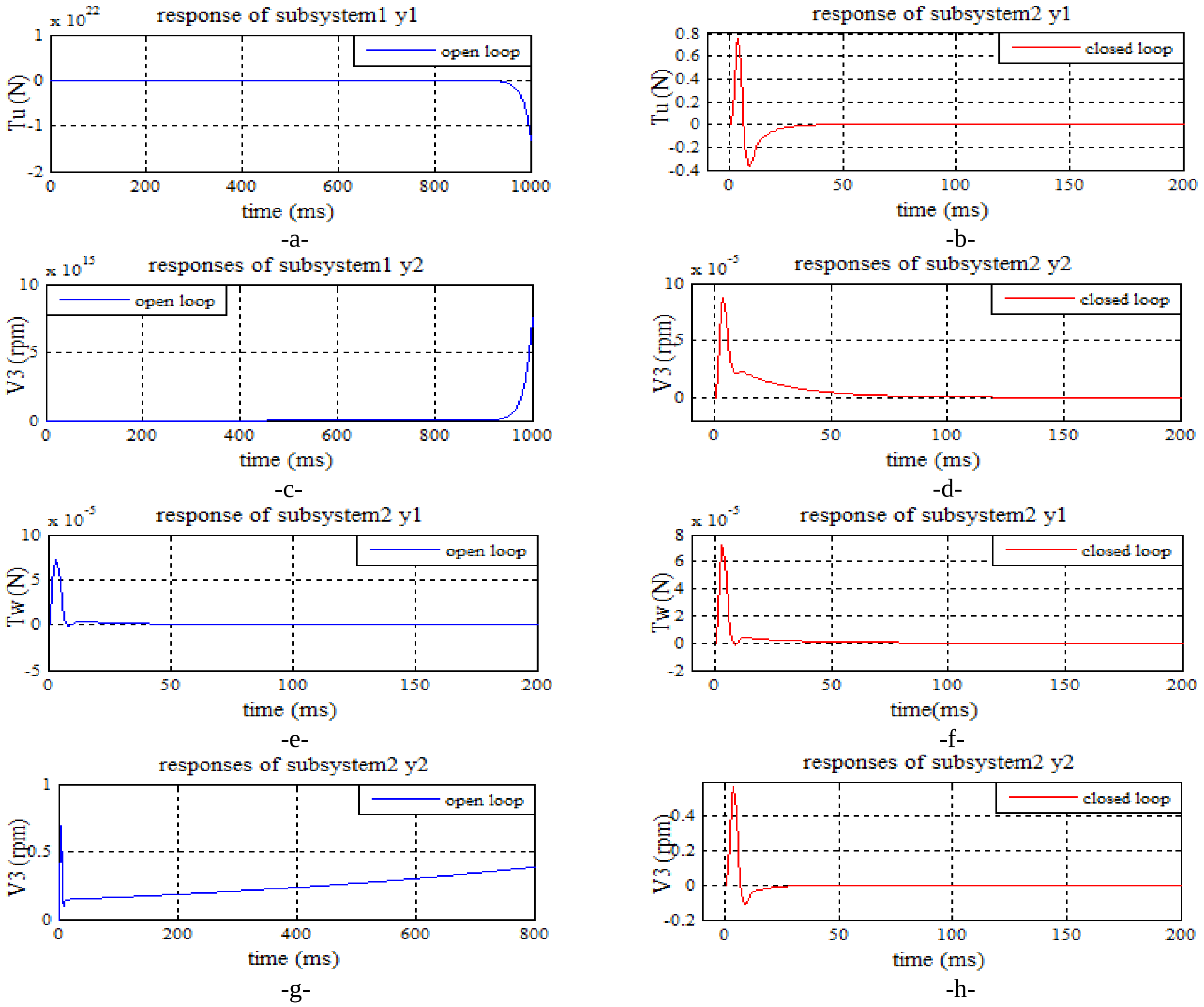

Fig.IV.26. Optimal decentralized responses for t_2; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2

***For subsystem1in Fig.IV.26**: unwinding motor is stabilized for $t = .2s$ with peak of 0.7 however the decomposed tractor motor response is stabilized at $t = 4s$ but peak of $8 * 10^{-6}$. The cost function is $J_1 = 1.15781 * 10^{14}$.

***For subsystem2 in Fig.IV.26**: winding motor response is stabilized in 0.2s with peak of $0.7 * 10^{-5}$ at .05s, however the decomposed tractor motor response is stabilized in 0.02s too but with peak of 0.6 as in non-optimal system, the cost function is $J_2 = 455.398$.

In comparing with optimal centralized control which gives cost function of $J_t = 1.21185 * 10^{14}$, it is clearly that $J_1 + J_2 \prec J_t$; which means that overlapping decomposition doesn't just minimize complexity of calculation but also minimize the cost function of the optimal system.

❖ For t_3

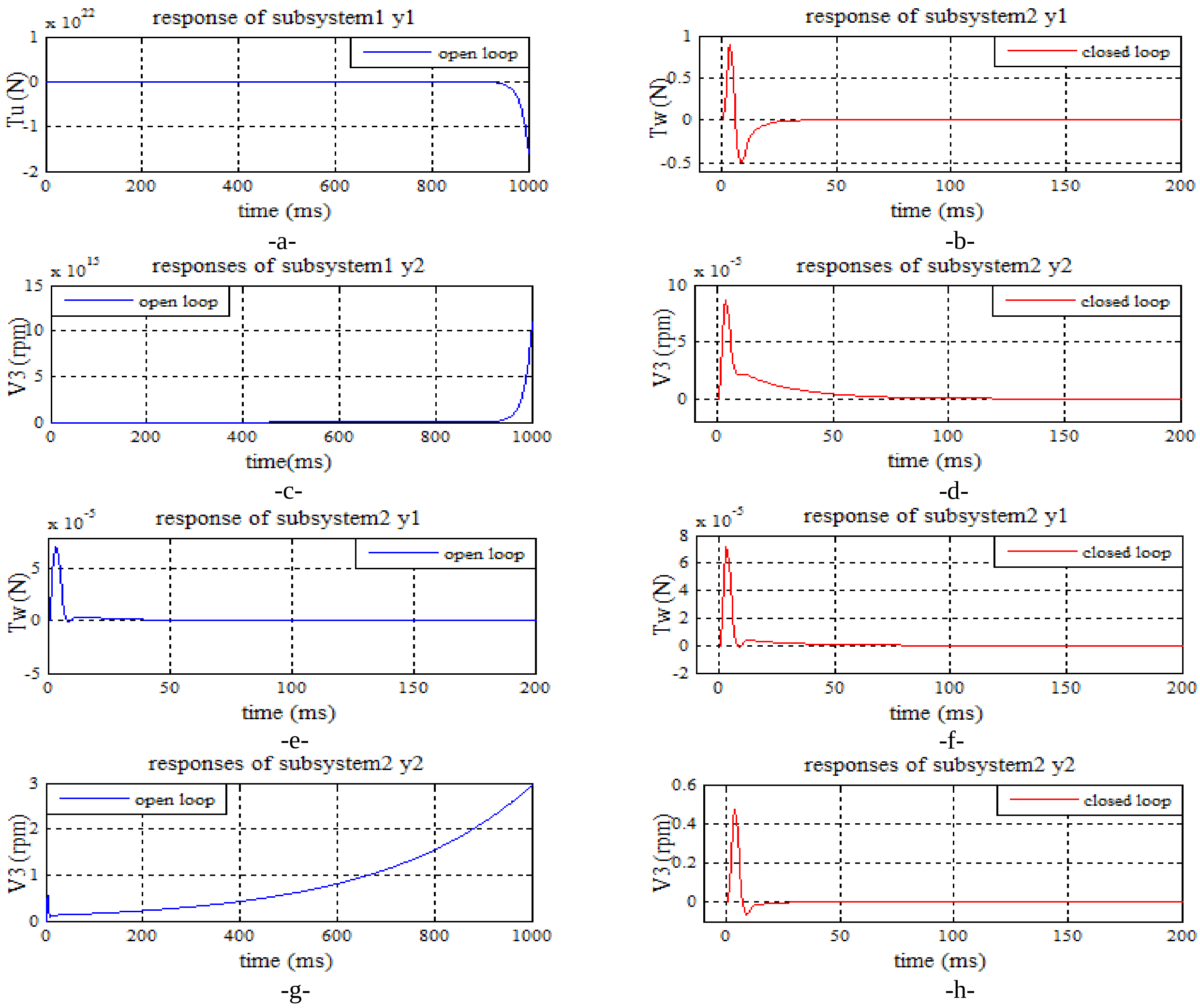

Fig.IV.27. Optimal decentralized responses for t_3 ; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2.

***For subsystem1in Fig.IV.27**: unwinding motor is stabilized for $t = .2s$ with peak of 1 however the decomposed tractor motor response is stabilized at $t = .8s$ but with peak of $8*10^{-5}$. The cost function is $J_1 = 1.0418*10^{14}$.

***For subsystem2in Fig.IV.27**: winding motor response is stabilized in .2s with peak of $0.7*10^{-5}$ at .05s, however the decomposed tractor motor response is stabilized in 0.2s too but with peak of 0.5 as in non-optimal system, the cost function is $J_2 = 509.955$.

In comparing with optimal centralized control which gives cost function of $J_t = 1.11383*10^{14}$, it is clearly that $J_1 + J_2 \prec J_t$; which means that overlapping decomposition doesn't just minimize complexity of calculation but also minimize the cost function of the optimal system.

- For t_4

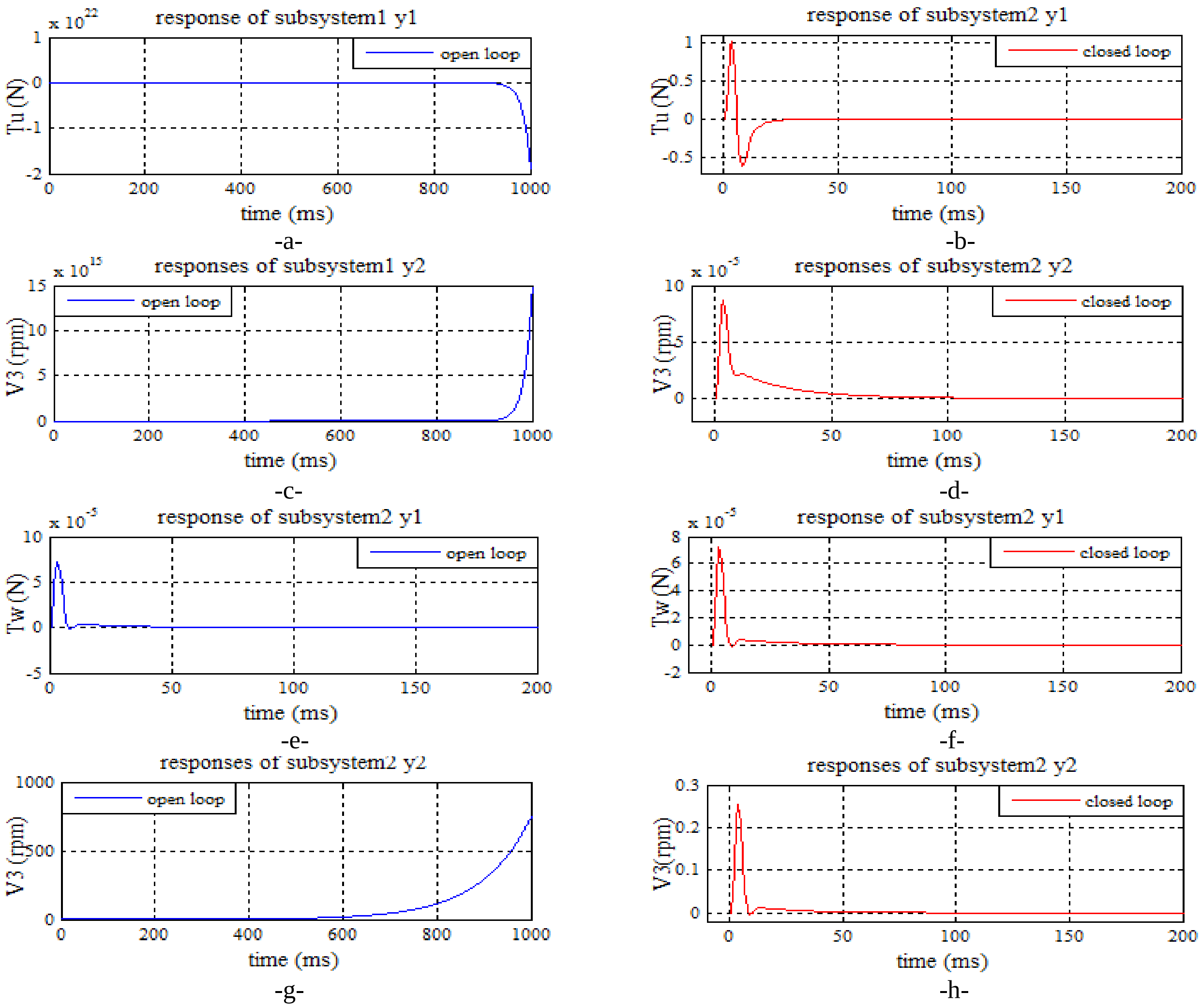

Fig.IV.28. Optimal decentralized responses for t_4; a, b, c, d: subsystem 1. e, f, g, h: subsystem 2.

*For subsystem1in Fig.IV.28**: unwinding motor is stabilized for $t = .2s$ with peak of 1 however the decomposed tractor motor response is stabilized at $t = .6s$ but peak of $7*10^{-4}$. The cost function is $J_1 = 9.6574*10^{13}$.

***For subsystem2in Fig.IV.28**: winding motor response is stabilized in 0.2s with peak of $7*10^{-5}$ at .05s, however the decomposed tractor motor response is stabilized in 0.2s too but with peak of 0.25, the cost function is $J_2 = 823.58$.n comparing with optimal centralized control which gives cost function of $J_t = 1.10809*10^{14}$, it is clearly that $J_1 + J_2 \prec J_t$; which means that overlapping decomposition doesn't just minimize complexity of calculation but also minimize the cost function of the optimal system.

We notice here that time response of optimal system is less than the non-optimal system except for t_0

But the sum of the cost function of subsystem is always less than the cost function of the original system; this is also one of the advantages of the overlapping decomposition technique for optimization principle.

V.4. Summary

In this chapter, we started by giving the algorithms that implemented in this project, then we move to describing MATLAB's graphical user interface and how to create it and use it , we also

explain how to use GUI that was implemented in order to represent the developed algorithms for each system (building system and web winding system).
We have finished by analyzing the results founded for each system and extracting the advantage and disadvantage of decentralized optimization.

Conclusion

Our objective in this study was to design decentralized optimal overlapping decomposition controller for real physical systems, for which we demonstrate the advantages of using overlapping decomposition techniques in minimizing the complexity of large scale system and also efficacy of decomposition of optimal control into smaller optimization problems.

The overlapping decomposition method has been presented in this study, and then the inclusion principle has been introduced to provide a mathematical framework for decentralized control. The inclusion of the cost function has also been discussed to incorporate the optimal control problem for large scale systems where the concept of contractibility of controllers has been discussed.

Optimal decentralized dynamic output feedback controllers design has been proposed for six floor building system and then three motors web winding systems with overlapping structure. Non-optimal overlapping centralized and decentralized controllers are designed for which we found that overlapping decentralized controller give better results in addition to simplifying calculation and reducing complexity.

For the building system, it contains six floors where this system is considered a combination of two overlapped subsystems for which subsystem one contains floors (1,2,3 and 4) and subsystem two contains floors (3,4,5 and 6), it is clear that floors (3,4) are the overlapped region between the two subsystems.

For the application of web winding system; it contains three motors (the unwinding motor, the tractor motor and the winding motor) and considered as two subsystems combination too, the first subsystem gathers the unwinding motor with the tractor motor, the second subsystem gathers the tractor motor with the winding motor, where it is clear here too that the overlapped region is the tractor motor, for the unwinding and the winding motor we are interested on the measuring the tensions (T_u and T_w).

Furthermore to improve these results we developed an optimization technique that allow us not just design optimal controller but also minimize the cost function of the whole system in comparison to original one. Where, for decomposed decentralized subsystems one and two J_1 and J_2 respectively is generally less than centralized cost function J_t.

Perspectives

Several assumptions topics made in this work may be questioned for further study

- Larger systems may be studied, especially where there more than one overlapped region in the system (multi-overlapping structure).

- Since the system studied around equilibrium point can't allow as guess stability far away from these points, it is necessary to more general method to conclude about it.

- This method we studied a linearized system which means our results depends not just in algorithm proposed but also in the linearization algorithms applied.

- Overlapping decomposition deals only with gain matrices for linear or linearized large scale system, it is necessary to discuss another method of decomposition deal with nonlinear system.

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Appendix A: Matrix Algebra

The purpose of this appendix is to present specific types of matrices that were used throughout the report. Mainly, the importance is given to the M-matrices, due to their usefulness in providing connective stability of large scale dynamic systems. The following notation will be used throughout this appendix.

$A=(a_{ij})$ denotes the real $n \times n$ matrix with a_{ij} being its i, j^{th} element and $i, j \in N$, where N is the set of indices $\{1, 2, ..., n\}$. A^T is the transpose of A, det(A) is its determinant. $\lambda_i(A)$ denotes the i^{th} Eigen values of A respectively.

❖ **Matrix rank**

The rank of a matrix is the dimension of the largest array with nonzero determinant.

❖ **Matrix Eigen values**

The Eigen values $\lambda_i[A](i=1,2,...,n)$ of an $n \times n$ matrix are defined to be real or complex numbers for which vectors X_I exist such that

$$AX_i = \lambda_i X_i, \; i=1,2,...,n \qquad \text{(A.1)}$$

❖ **Quadratic forms and definite matrices**

Given a square matrix Q and a vector $X \in R$, we call $f(x) = X^T QX$ a bilinear or quadratic form.
We assume Q is real

- Q is positive semi-definite if $f(x) \geq 0$ for all nonzero x.
- Q is positive definite if $f(x) \succ 0$ for all nonzero x.
- Q is negative semi-definite if $f(x) \leq 0$ for all nonzero x.
- Q is negative indefinite if $f(x) \prec 0$ for all nonzero x.

❖ **Norm of matrix**

Let be an $n \times m$ matrix; the scalar valued function $\|A\|$ is called the norm of A if it satisfies the following properties:

- $\|A\| \succ 0$ provided not all $a_{ij} = 0$.
- $\alpha\|A\| = |\alpha|\|A\|$ for all $\alpha \in R$.
- $\|A+B\| \leq \|A\| + \|B\|$

Some valid norms are:

$$\begin{cases} \|A\|_1 = \sum_{i=1}^{n}\sum_{j=1}^{m}|a_{ij}| \\ \|A\|_2 = (\sum_{i=1}^{n}\sum_{j=1}^{m}a_{ij}^2)^{\frac{1}{2}} \\ \|A\|_\infty = \max\sum_{j=1}^{m}|a_{ij}| \end{cases} \qquad \text{(A.2)}$$

Definition.A1: an $n \times n$ matrix A is said to be nonnegative (positive) if $a_{ij} \geq 0$ ($a_{ij} \succ 0$) for all $i, j \in N$,we denote it by $A \geq 0 (A \succ 0)$.

The Metzler matrix was first introduced in economics. The economists do not agree on the definition of the Metzler matrix, two definition are stated:

Definition.A.2: an $n \times n$ matrix A is Metzler matrix if

$$a_{ij} = \begin{cases} \prec 0, & i = j \\ \geq 0, & i \neq j \end{cases} \quad \text{(A.3)}$$

for all $i, j \in N$.

Definition.A.3: an $n \times n$ matrix A is Metzler matrix if $a_{ij} \geq 0,\ i \neq j$, where $i, j \in N$, these two definition are used for stability tests.

Definition.A.4: an $n \times n$ matrix A is called stable (or Hurwitz) if $\mathrm{Re}(\lambda_i(A)) \prec 0$ for all $i \in N$.

Theorem .A.1:

An $n \times n$ Metzler matrix A is stable if and only if:

$$(-1)^K \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & \dots & \dots & \dots \\ a_{k1} & a_{k2} & \dots & a_{kk} \end{vmatrix} \succ 0 \quad \forall \mathrm{k} \in \mathrm{N} \quad \text{(A.4)}$$

Another fundamental theorem related to Metzler matrices is:

Theorem.A.2:

An $n \times n$ Metzler matrix A is stable if and only if it is quasi-dominant negative diagonal.

We can dispense with the Metzler structure of A and use the diagonal dominance to conclude stability.

Another class of matrices is useful when using vector Lyapunov functions in establishing stability of large scale systems is considered.

Definition A.5: a real $n \times n$ matrix A is called positive definite if $x^T Ax \succ 0$ for all n vector $x \neq 0$.

by writing $A = B + C$, where $B = (A + A^T)/2$ is the symmetric and $C = (A - A^T)/2$ the Skew symmetric part of A, we can show that $x^T Cx = (x^T Cx) = -x^T Cx$ and $x^T Cx = 0$, so that $x^T Ax = x^T Bx$. Therefore, A is positive definite if and only if its symmetric part $B = (A + A^T)/2$ is positive definite. We can also recall the classical Lyapunov results:

Theorem A.3

A matrix A is stable if and only if for any positive definite symmetric matrix G there is a positive definite symmetric matrix H such that

$$A^T A + HA = -G \quad \text{(A.5)}$$

In stability analysis of large scale systems, it is essential to find conditions under which is the solution H of the Lyapunov matrix equation (A.3) is a positive matrix, that is all its elements are positive numbers. This problem is resolved as follows:

Theorem A.4:

If a matrix A is Metzler and stable, then for any positive and positive definite symmetric matrix G there is a positive and positive definite symmetric matrix H as a solution of the corresponding Lyapunov matrix equation.

Conjecture

If a matrix A is stable, then there is positive definite symmetric matrix G such that the matrix H as a solution of the corresponding Lyapunov matrix equation is a positive and positive definite symmetric matrix.

- **Pseudo-inverse**:
  - **left invertible transformations:**

Let us denote by $S(S_1, S_2)$ the set of all linear transformations mapping the linear space S_1 into the linear space S_2 over the field E, both S_1 and S_2 are assumed to be finite dimensional.

A transformation $T \in S(S_1, S_2)$ is said to be left invertible if there exist a transformation T_1 such that:

$$T_1 T = I_1 \quad \text{(A.6)}$$

Where I_1 denotes the identity transformation in S_1. If equation (A.4) holds, then the transformation T_1 is called a left inverse of T (pseudo-inverse).

Theorem A.5:

Let $T \in S(S_1, S_2)$ and let $\dim(S_1) = n, \quad \dim(S_2) = m$ the following statements are equivalent:

1. T is left invertible.
2. $KerT = \{0\}$.
3. T defines an isomorphism between S_1 and $\operatorname{Im} T$
4. $m \geq n$ and $rank(A) = n$.
5. Every representation of T is an $m \times n$ matrix with $m \geq n$ and full-rank.

- **left invertible matrices**

Theorem A.6:

The followings statements are equivalent for any matrix $A \in E^{m\times n}$

1. The matrix A is left invertible.
2. $m \geq n \;\; and \;\; rank(A) = n$.
3. The columns of A are linearly independent as members of E^m.
4. $\ker A = \{0\}$.

The set of all left inverses of A are given by the solutions of

$$XA = I_n \tag{A.7}$$

Where A is left invertible. To describe the solutions of equation above, we first apply a permutation matrix $P \in E^{m\times m}$ such that the $n \times m$ nonsingular sub-matrix A_1 in PA occupies the top n rows, that is:

$$PA = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}: \;\; A_1 \in E^{n\times m}, \;\; \det A_1 \neq 0 \tag{A.8}$$

It is easily verified that matrices of the form

$$\begin{bmatrix} A_1^{-1} - BA_2A_1^{-1} & B \end{bmatrix} P \;\; \text{for any } B \in E^{n\times(m-n)} \tag{A.9}$$

Satisfy equation above, that are left inverses (Pseudo-inverses) of A.

Appendix B: interaction analysis of MIMO system

Consider the multi input multi output system defined by the following equation:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad \text{(B.1)}$$

Where $x(t) \in R^n$, $y(t) \in R^l$ and $n, l \succ 1$; this means that there is interaction between the loops of the system, analyzing such interaction can be done as follows:

1) Analysis of the interaction by constructing the **DRGA**
2) Selection of the preferable control configuration.
3) Design of compensator $k(s)$ using proposed algorithm**.**

System described by eq (B.1) can be written in Laplace domain as follows:

$$\frac{y(s)}{x(s)} = G(s) \qquad \text{(B.2)}$$

where $$G(s) = C(SI - B)^{-1} \qquad \text{(B.3)}$$

Analysis of interaction for this system can be done by studying $G(s)$ in specific value of frequencies, where the Laplace variable is

$$S = j\omega \qquad \text{(B.4)}$$

Considerable insight into the behavior of interacting systems under perfect control is afforded by the RGA. For a system of interacting loops the general element of the RGA can be written as

$$\Lambda = (\lambda_{ij}) \qquad \text{(B.5)}$$

where

$$(\lambda_{ij}) = \frac{(\frac{\partial y_i}{\partial u_j})_{\text{all loops open}}}{(\frac{\partial y_i}{\partial u_j})_{\text{only } y_i - u_j \text{ open}}} \qquad \text{(B.6)}$$

The partial derivatives represent steady state gain factors evaluated under the conditions noted in parenthesis (the array therefore relates to the use of perfect controllers only). For a system of two loops application of the property that elements in any row or column in Λ sum to unity allows construction of the full array on the basis of (λ_{11}) only. Having constructed Λ , an extensive body of experience in its interpretation can be drawn upon. In essence, diagonal dominance in Λ with diagonal elements close to unity indicates both correct pairing of measured and manipulated variables and the absence of potential for interaction. The RGA has been extended to take account of system dynamics in the DRGA [46].