People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research

University M'Hamed BOUGARA – Boumerdes

Institute of Electrical and Electronic Engineering

Department of Power and Control Engineering

Final Year Project Report Presented in Partial Fulfilment of
the Requirements for the Degree of

MASTER

In **Control Engineering**

Option: **Control Engineering**

Title:

Simulation of Robust Controllers for Disturbance Rejection in Quad-copter

Presented by:

- **BELKHEIR Rabia**

Supervisor:

- **BOUSHAKI Razika**

Dedication

This thesis is dedicated to the sake of Allah, my Creator and my Master; my great teacher and messenger Mohammed – May peace be upon him- who taught us the purpose of life; to our brothers and sisters in Palestine; and to all the Algerian martyrs, the symbol of sacrifice.

To my dear father, who has been nicely my supporter until the end, and who taught me the value of hard work; my beloved mother for encouraging me all the way and motivating me; my sisters and brothers who supported me throughout the process .Without forgetting my friends, who uplift me and inspire me; and all the kind people in my life.

Acknowledgement

I am grateful to all the people without whom this work would not have been achieved. Thank you to my advisor Pr.Boushaki for giving the opportunity to do this project and for her guidance. Also, thank you to Dr.Abdellah Mokhtari teacher at USTO university Oran and Mr.Amine Boukhalfa for clarifying some confusions and providing the necessary tools for the project completion.

Abstract

In this work kinematics and dynamics of a quad-copter were presented, the quad-copter is studied in two frames: inertial or earth frame and body frame, the first is attached to the ground and the latter is attached to the aircraft's center. Forces and moments acting on the drone were studied as well as aerodynamic effects and rotor dynamics. From this information, a state space model was derived which was used to design three controllers to stabilize the system. When building the model the quad-copter and propellers are assumed to be rigid.

One controller was a PID and the others followed Backstepping and Gain-scheduling methods. PID is a linear controller that minimizes the error between actual and desired states through proportional; integral and derivative terms. Backstepping is a method developed for nonlinear systems that can be decomposed into smaller subsystems and then controlling inputs are derived based on Lyapunov theory of stability. As for Gain-scheduling, it is an expansion of the PID method to be suitable for nonlinear systems where multiple PID controllers are used, switching between controllers is based on a scheduling variable, in this thesis the scheduling variable was chosen to be the desired state. In all these controllers parameters are tuned using repetitive trials until satisfactory outcome is achieved.

In the end, simulation results were presented and discussed. Additionally, advantages and disadvantage of each method are highlighted to conclude with a comparison between the three controllers.

Table of Contents

Abstract I

Table of Contents II

List of Acronyms IV

List of symbols IV

List of figures V

List of tables VI

General Introduction VII

Chapter 01: Modeling of Quad-rotor 1

1.1 Kinematic Model 2

1.2 Dynamic Model 3

1.2.1 Rotor Dynamics 4

1.2.2 Forces and Moments Acting on the Drone 5

1.2.3 Possible Drone Motion 7

1.2.4 Inertia Matrix 8

1.2.5 Translational Equations of Motion 8

1.2.6 Rotational Equations of Motion 9

1.2.7 State Space Model 9

Chapter 02: PID Control of Quadrotor 12

2.1 Choice of model parameters 13

2.2 PID method 14

2.3 PID Parameter Tuning 14

2.4 PID Method Results 16

2.4.1 Response without Disturbance 16

2.4.2 Response with Disturbance 18

Chapter 03: Backstepping Control Of Quad-rotor 20

3.1 Roll Controller 21

3.2 Pitch Controller 23

3.3 Yaw Controller 24

3.4 Altitude Controller 24

3.5 Backstepping Method Results 26

3.5.1 Response without Disturbance 26

3.5.2 Response with Disturbance 28

Chapter 04: Gain-scheduling Control of Quad-rotor 30

4.1 Gain-scheduling Method 31

4.2 Gain-scheduling Method Results 33

4.3 Comparison between the Three Developed Controllers 35

Conclusion 36

References 37

List of Acronyms

3D	Three dimensions
DC	Direct current
CW	Clockwise
CCW	Counter clockwise
PID	Proportional integral derivative controller

List of symbols

ϕ	Roll angle
Θ	Pitch angle
Ψ	Yaw angle
R^{E}_{B}	Rotation matrix Earth to body frame
F	Force
M	Torque
J	Quad-copter inertia
Ω	Rotor speed
Jr	Rotors inertia

List of figures

Fig. 1.1: Body and Earth frames. …….…….. 2

Fig. 1.2: DC motor schematic diagram …….……..…… 4

Fig. 1.3: Forces and moments acting on the quad-rotor. …… 6

Fig. 1.4: Drone possible motions. ……...…… 7

Fig2.1: PID controller …… 14

Fig. 2.2: PID controller tuning …….……..… 14

Fig. 2.3: PID response with no disturbance. ……..…… 16

Fig. 2.4: Effective inputs without disturbance …….……..…… 17

Fig. 2.5: Quad-copter trajectory ……...……..… 17

Fig. 2.6: PID response with disturbance ……...…..…… 18

Fig. 2.7: Effective inputs witt disturbance. ……..……...…… 19

Fig. 3.1: Backstepping controller……..…… 25

Fig. 3.2: Backstepping response with no disturbance ……...…..……..…… 26

Fig. 3.3: Effective inputs without disturbance ……...…… 27

Fig 3.4: Backstepping response with disturbance ……....…… 28

Fig 3.5: Effective inputs with disturbance ……....…… 29

Fig 4.1: Altitude response ……..…… 33

Fig 4.2: Roll response …… 34

Fig 4.3: Pitch response ……..…… 34

Fig 4.4: Pitch response ……..……..……. 35

List of tables

Table 2.1: The physical parameters of the quad-copter. ……..……..…… 13

Table 2.2: PID parameters for altitude control. ……..…..……. 15

Table 2.3: PID parameters for altitude control. ……. 15

Table 4.1: Gain-scheduling PID parameters for altitude control. ……..…… 31

Table 4.2: Gain-scheduling PID parameters for roll control. ……..……..…….. 32

Table 4.3: Gain-scheduling PID parameters for pitch control. …….…….. 32

Table 4.4: Gain-scheduling PID parameters for yaw control. ……...……. 33

General Introduction

A quad-copter is an aircraft with four rotors, it is much smaller and cheaper to make than a helicopter, it is easier to maintain and manufacture, another advantage is the absence of a tale.

The first quad-copter to be made dates back to 1908 by Louis Breguet, since then the need for smaller aircrafts with high moving abilities has increased the interest in researching multi rotor drones.

The thrust produced by the rotors enables the quad-copter to fly, the reason to have four rotors is so that two of them rotate in a CW direction and the other two rotate in the opposite direction, like that the torque produced will be cancelled and there is no need for a tale. The upward force produced by rotors one through four must be greater than the aircraft weight to ascend, moving forward, backward and sideways is achieved by rotating the quad-copter about x and y axes, and rotation about the z-axis occurs when there is a difference in torques that are produced by the rotors spin, thus enabling the quad-copter to enjoy a 6 DOF motion.

The drone has application in multiple fields like: agriculture, journalism, natural disaster, submarine and military, making this research area versatile and interesting. The aim of this work is to test several methods to control the quad-copter and make a comparative study between them.

In this thesis, chapter one is dedicated to studying the quad-copter's kinematics and dynamics, then a mathematical model was derived, altitude and roll angles were chosen to be the state space variables and the equations were obtained from the kinematics and the dynamics. This provided the basis for the simulation of the aircraft which then was build using Simulink, then three control methods were use to stabilize it.

In chapter two, the first method is tested which is the PID, the PID controller is one of the most used controllers because of its ease of understanding and implementation, it is based on controlling the error between actual and desired state, where three parts of control are present: proportional, integral and differential controllers, the proportional term acts on error where as the differential term act on its rate of change parameters were tuned using trial and error until satisfactory control was reached; the control system was tested with and without disturbance over a specified trajectory.

In chapter two the control inputs were derived according to the Backstepping method then tested on the drone model under no disturbance as well as when the drone is under external disturbance. Backstepping method uses Lyapunov stability to acquire the inputs through mathematical manipulation, it was first developed by Petar V. Kokotovic and others in 1990.

The fourth chapter discusses Gain-scheduling which is an extension of the PID controller to suit the nonlinear nature of the system by using multiple PIDs across multiple operating points and switch between them. After that, these methods are compared between each other to conclude that the PID is only suitable for near equilibrium use where as the other methods perform good control across all ranges of states.

Research on quad-copter merges field like mechanic and control engineering and provide a good background for further research in robotics. Moreover, this field is relatively new so it is exiting to contribute to the collection of research done on drones.

This thesis provides a comparison between multiple methods of control after testing them through a computer simulation. It is noted that PID even though more suitable for linear plants, it gave satisfactory outcome near equilibrium. However, it wasn't able to control the quad-copter for all values of states, in the opposite hand, Backstepping controller is suitable for this control system, since it was developed to account for plant's nonlinearities. Gain-scheduling compensate for the linear nature of the PID by using multiple controllers each one for a specific operation point, so it is suitable for nonlinear plants but it had spikes in the control inputs.

Chapter 01

Modeling of Quad-rotor

In this chapter, a mathematical model for the quad-rotor is derived based on the Newton-Euler laws. The following assumptions have been made:

- The structure of the quad-rotor is rigid and symmetrical.
- The center of mass of the quad-rotor coincides with the body fixed frame origin.
- The propellers are rigid.
- The trust and drag are proportional to the square of propeller's speed.

The kinematic and dynamic models are developed, the aerodynamics effects acting on the quad-rotor is discussed as well as the rotor dynamics of the actuators. Finally, a state space model is presented which later is used for testing the control methods.

1.1 Kinematic Model

The kinematic model describes the motion of objects in space, it is noted that the quad-copter has 6 degrees of freedom so it can rotate around the three principal axes of its body, which are as follow: the x-axis goes from the center of the aircraft and along one of arms; y-axis goes through the center and is aligned with the arm perpendicular to the previous one and the z-axis passes through the center and is perpendicular to the other two axes and goes downward.

Rotation about the x-axis is called roll and the angle is denoted ϕ, similarly rotation about the y-axis is labeled pitch with angle θ, and for the z-axis the rotation is denoted yaw and the angle is ψ. ϕ, θ and ψ are the Euler angles that describe the motion in 3D space.

X, Y and Z are the axes constituting the body frame that is attached to the aircraft; on the other hand the Earth frame is attached to the ground with axes e1, e2, e3.

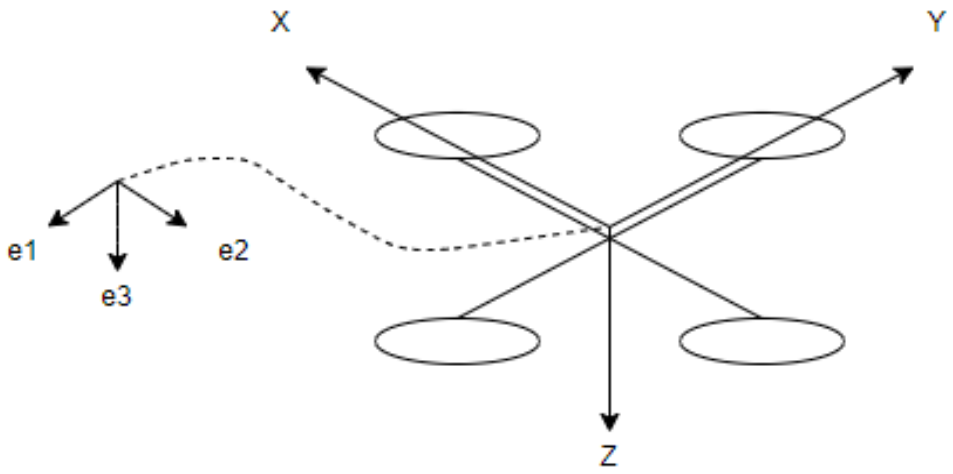


Fig. 1.1: Body and Earth frames.

Conversion from Earth to body frame, three successive rotations are made: yaw-pich-roll which are summed in the rotation matrix $R^{E}_{B.}$

• Rotate about the z-axis

$$R\psi = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1.1)$$

• Rotate about the new y-axis.

$$R\theta = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \qquad (1.2)$$

• Rotate about the new x-axis

$$R\varphi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\varphi) \end{bmatrix} \qquad (1.3)$$

$$R^{E}_{B} = R\psi\, R\theta\, R\varphi \qquad (1.4)$$

$R^{E}_{B}=$

$$\begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \sin(\varphi)\sin(\theta)\cos(\psi) - \cos(\varphi)\sin(\psi) & \sin(\varphi)\sin(\theta)\sin(\psi) + \cos(\varphi)\cos(\psi) & \sin(\varphi)\cos(\theta) \\ \cos(\varphi)\sin(\theta)\cos(\psi) + \sin(\varphi)\sin(\psi) & \cos(\varphi)\sin(\theta)\sin(\psi) - \sin(\varphi)\cos(\psi) & \cos(\varphi)\cos(\theta) \end{bmatrix} \qquad (1.5)$$

The transpose of the above matrix is used to return to Earth frame since rotation matrices are orthonormal.

1.2 Dynamic Model

To understand the behavior of the aircraft, the physics governing its motion are presented in this section. Forces and moments contribute in the deriving of equations of motions, two equations are distinguished: translational and rotational equations of motions, the first governs motions along x, y and z axes while the ladder governs roll, pitch and yaw motions.

1.2.1 Rotor Dynamics

For most drone applications brushless DC motor is used since it provides high power for its weight, it uses DC current and has a magnetic core. The following figure presents the electric equivalent circuit of the DC motor:

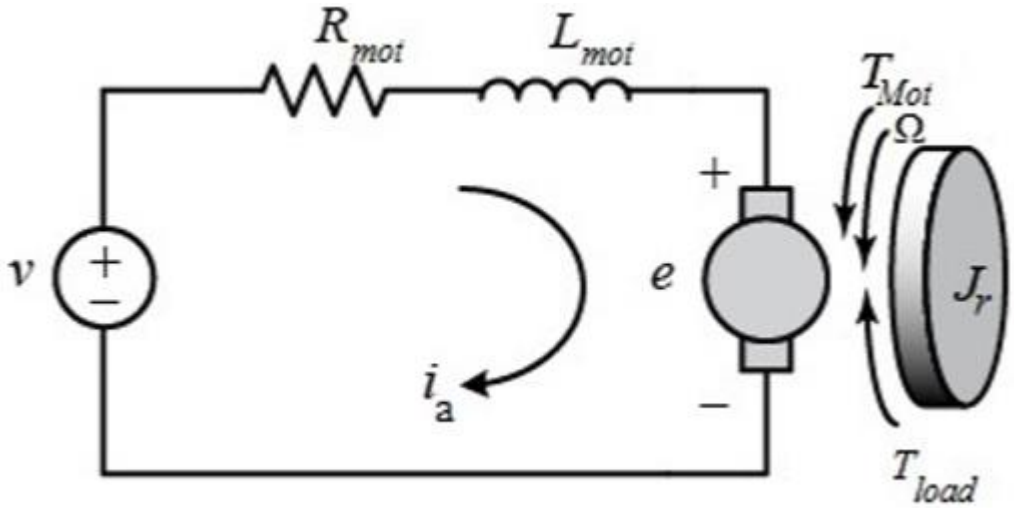


Fig. 1.2: DC motor schematic diagram

Using Kirchhoff's Voltage Law:

$$v = R_a i_a + L_a \frac{di_a}{dt} + e \quad (1.6)$$

$$e = k_e \Omega \quad (1.7)$$

$$J_r \dot{\Omega} = T_{mot} - T_{load} \quad (1.8)$$

$$T_{mot} = K_T i_a \quad (1.9)$$

$$T_{load} = K_M \Omega^2 \quad (1.10)$$

Replacing (1.7) in (1.6) and (1.9), (1.10) in (1.8):

$$v = R_{mot} i_a + K_{mot} \Omega_i \quad (1.11)$$

$$J_r \dot{\Omega}_\iota = K_{mot} i_a - K_M \Omega_i^2 \quad (1.12)$$

Combining the two previous equations yield:

$$v = \frac{R_{mot}}{K_{mot}} J_r \dot{\Omega}_\iota + K_{mot} \Omega_i + K_M R_{mot} \Omega_i^2 \quad (1.13)$$

For steady state rotation speed can be related to input voltage using:

$$\Omega = K_T \, v \quad (1.14)$$

1.2.2 Forces and Moments Acting on the Drone

In addition to the gravitational force, there are aerodynamic forces that act on the airfoil. These forces are the lift and drag and are given by:

$$F_i = \frac{1}{2} \rho A C_T r^2 \Omega_i^2 \quad (1.15)$$

$$D_i = \frac{1}{2} \rho A C_D r^2 \Omega_i^2 \quad (1.16)$$

Where:

- ρ : air density
- A : blade area
- C_D, C_T : aerodynamic coefficients
- r : the radius of blade
- Ω_i : angular velocity of rotor i

The four propellers exude a lifting force

$$F_i = K_f \Omega_i^2 \quad (1.17)$$

These forces produce moments about the x and y axes

$$M_x = -F_2 l + F_4 l \quad (1.18)$$

$$M_x = l K_f (-\Omega_2^2 + \Omega_4^2) \quad (1.19)$$

Similarly:

$$M_y = l K_f (\Omega_1^2 - \Omega_3^2) \quad (1.20)$$

In addition to the upward force, the rotating propeller also produces opposite moments. For example, a rotor rotating in the CW direction will generate a torque that causes the drone body to rotate in the CCW direction. This torque is given by:

$$M_i = K_M \Omega_i^2 \quad (1.21)$$

The torque generated by the motor is opposite to its rotating direction, so **CW** and **CCW** rotating motors generate opposite torque. This is why we have **CW** and **CCW** rotating motors, so that in a stable hovering state, the moments from the **2 CW** and **2 CCW** rotating rotors cancel each other out, and the drone will not keep rotating around its body axis

The overall moment generated around the drone's **z-axis** is given by the sum of all 4 moments. Rotor forces and moment are shown in **Fig 1.3**.

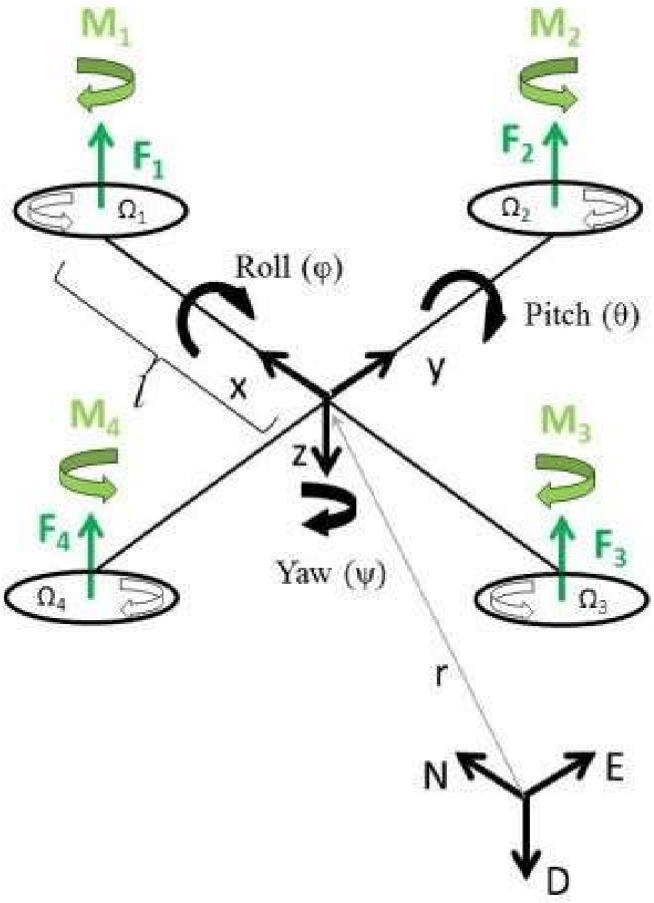


Fig. 1.3: Forces and moments acting on the Quad-rotor.

Additional forces and moments acting on the quad-copter are: gyroscopic moment, which is the tendency of rotating object to reject changes in its axis of rotation; and drag force that is when the quad-copter is moving, air friction will produce a resisting force given by:

$$F_a = K_t \dot{r} \qquad (1.22)$$

- K_t is a constant matrix called the aerodynamic translation coefficient matrix a
- $\dot{r}$ is the derivative of the position vector r.

This force will in turn produce a drag moment as follows:

$$M_a = K_r \dot{\eta} \qquad (1.23)$$

- K_r is a constant matrix called the aerodynamic rotation coefficient matrix and
- $\dot{\eta}$ is the Euler rate.

1.2.3 Possible Drone Motion

The overall motion of the quad-copter could be divided into multiple rotations and translations. Possible motions are demonstrated in **Fig 1.2.**

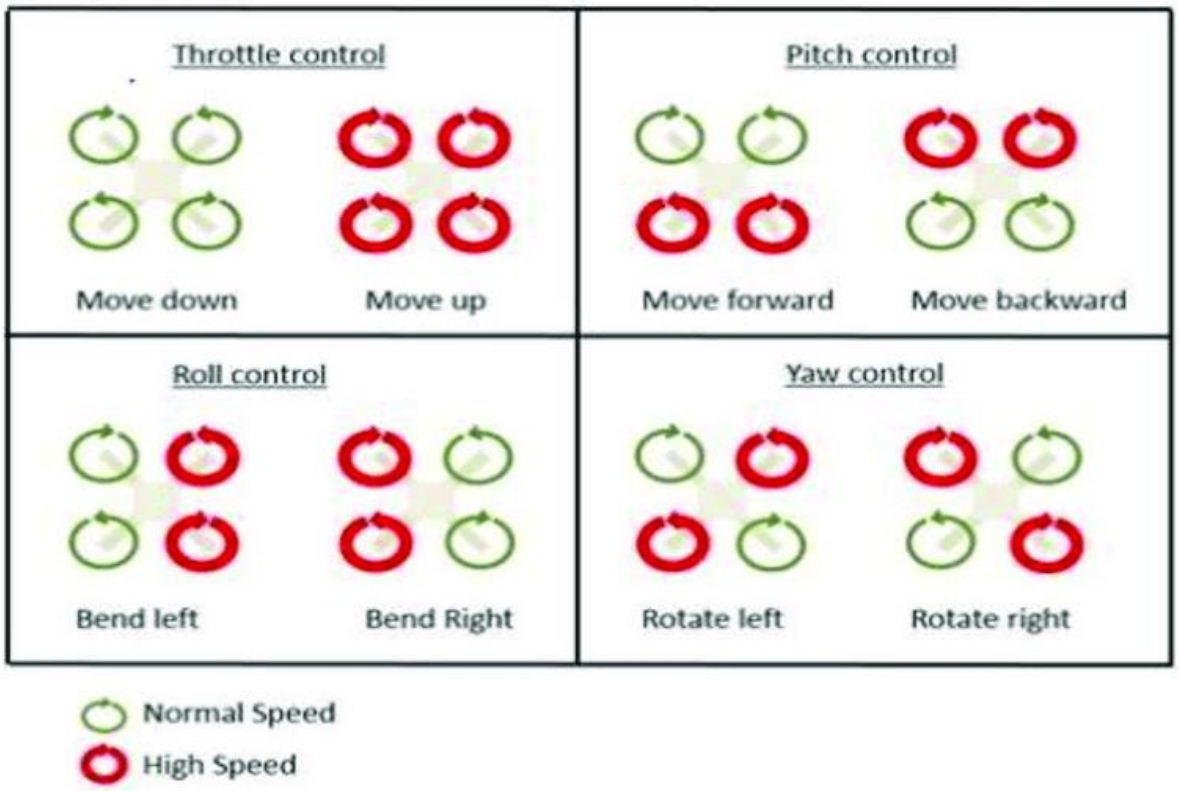


Fig. 1.4: Drone possible motions.

For motion in the **z-axis** the thrust from the four rotors cancels the aircraft weight in hover, to move upward the resultant force of the rotor must be greater than the weight, and similarly if the weight is greater than the resultant thrust the drone will drop. Roll and pitch slides the quad-copter forward, backwards or sideways, to move to the left for example F2 and F3 must be greater than F1 and F4, on the other hand to pitch F4 and F3 exceed F1 and F2, and if two motors that spin in the same direction have higher speed than the other two the quad-copter will rotate in their direction.

1.2.4 Inertia Matrix

The inertia matrix is given by:

$$J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \qquad (1.24)$$

Due to the symmetry product of inertia element is zero;

- **Ixx**, **Iyy** and **Izz** are moments of inertia about the principle axes in the body frame.

1.2.5 Translational Equations of Motion

The Newton's second law is used to derive the translational equations of motion in the Earth frame.

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} - \mathrm{R}_{\mathrm{E}}^{\mathrm{B}} F + F_a \qquad (1.25)$$

Where

- $r = [x \quad y \quad z]^T$ Quad-rotor's distance from the inertial frame.
- m Quad-rotor's mass
- g gravitational acceleration $g = 9.81\ m/s^2$
- F Nongravitational forces acting on the Quad-rotor.
- Fa drag force.

$$F = \begin{bmatrix} 0 \\ 0 \\ K_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \end{bmatrix} \qquad (1.26)$$

The first two rows of the force vector are zeros because there is no forces acting on the x and y axes, the last row is simply an addition of the thrust forces produced by the four propellers. The negative sign is due to the fact that the thrust is upwards while the positive z-axis in the body framed is pointing downwards.

- $\boldsymbol{F}$ is multiplied by the rotation matrix $\mathrm{R}^{\mathrm{B}}{}_{\mathrm{E}}$ to transform the thrust forces of the rotors from the body frame to the inertial frame.

1.2.6 Rotational Equations of Motion

Applying Newton-Euler method in the body frame :

$$J\dot{\omega} + \omega \times J\omega + M_G = M - M_a \qquad (1.27)$$

Where

- J Quad-rotor's diagonal inertia matrix.
- ω angular body rates.
- M_G gyroscopic moments due to rotors' inertia.
- M moments acting on the Quad-rotor in the body frame.
- M a drag moment.
- $M_G = [0 \quad 0 \quad J_r\Omega_r]^T \qquad (1.27)$
- $\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \ (1.28)$

So the rotational equation of motion becomes:

$$J\dot{\omega} + \omega \times J\omega + \omega \times [0 \quad 0 \quad J_r\Omega_r]^T = M_B - M_a \qquad (1.28)$$

1.2.7 State Space Model

Using the equations of motions previously derived, a state space model is obtained. First the state vector is defined as:

$$X = [x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12}]^T \qquad (1.29)$$

$$X = \left[\phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi} \ z \ \dot{z} \ x \ \dot{x} \ y \ \dot{y}\right]^T \qquad (1.30)$$

And the effective inputs as:

$$U = [U_1 U_2 U_3 U_4] \qquad (1.31)$$

Where

$$U_1 = K_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \qquad (1.32)$$

$$U_2 = K_f(-\Omega_2^2 + \Omega_4^2) \qquad (1.33)$$

$$U_3 = K_f(\Omega_1^2 - \Omega_3^2) \qquad (1.34)$$

$$U_4 = K_M(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \qquad (1.35)$$

It can be written in matrix form as:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} K_f & K_f & K_f & K_f \\ 0 & -K_f & 0 & K_f \\ K_f & 0 & -K_f & 0 \\ -K_M & K_M & -K_M & K_M \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \qquad (1.36)$$

U1 represents the resultant upward force produced by the four rotors, U2 and U3 are the difference in force on the x and y axes respectively, this difference is what produces torque about those axes, where as U4 is the sum of the rotating torques of the four rotors and is what causes rotation about the z-axis. From this equation (1.29) becomes:

$$\begin{bmatrix} I_{xx}\ddot{\phi} \\ I_{yy}\ddot{\theta} \\ I_{zz}\ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\theta} I_{zz}\dot{\psi} - \dot{\psi} I_{zz}\dot{\theta} \\ \dot{\psi} I_{xx}\dot{\phi} - \dot{\phi} I_{zz}\dot{\psi} \\ \dot{\phi} I_{yy}\dot{\theta} - \dot{\theta} I_{zz}\dot{\phi} \end{bmatrix} + \begin{bmatrix} \dot{\theta} J_r \Omega_r \\ -\dot{\phi} J_r \Omega_r \\ 0 \end{bmatrix} = l \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} \qquad (1.37)$$

Rewriting to have the angular acceleration in term of the other variables:

$$\ddot{\phi} = \frac{l}{I_{xx}} U_2 - \frac{J_r}{I_{xx}} \dot{\theta}\Omega_r + \frac{I_{yy}}{I_{xx}} \dot{\psi}\dot{\theta} - \frac{I_{zz}}{I_{xx}} \dot{\theta}\dot{\psi} \qquad (1.38)$$

$$\ddot{\theta} = \frac{l}{I_{yy}} U_3 - \frac{J_r}{I_{yy}} \dot{\phi}\Omega_r + \frac{I_{zz}}{I_{yy}} \dot{\phi}\dot{\psi} - \frac{I_{xx}}{I_{yy}} \dot{\psi}\dot{\phi} \qquad (1.39)$$

$$\ddot{\psi} = \frac{l}{I_{zz}} U_4 + \frac{I_{xx}}{I_{zz}} \dot{\theta}\dot{\phi} - \frac{I_{yy}}{I_{zz}} \dot{\phi}\dot{\theta} \qquad (1.40)$$

By defining the following constants:

$$a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}$$

$$a_2 = \frac{J_r}{I_{xx}} \qquad b_1 = \frac{l}{I_{xx}}$$

$$a_3 = \frac{I_{zz} - I_{xx}}{I_{yy}} \qquad b_2 = \frac{l}{I_{yy}}$$

$$a_4 = \frac{J_r}{I_{yy}} \qquad b_3 = \frac{l}{I_{zz}}$$

$$a_5 = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

Using the definitions above, equations (1.38) through (1.40) can be written as:

$$\ddot{\phi} = b_1 U_2 - a_2 x_4 \Omega_r + a_1 x_4 x_6 \quad (1.41)$$

$$\ddot{\theta} = b_2 U_3 + a_4 x_2 \Omega_r + a_3 x_2 x_6 \quad (1.42)$$

$$\ddot{\psi} = b_3 U_4 + a_5 x_2 x_4 \quad (1.43)$$

As for the transitional equation of motion:

$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -mg\end{bmatrix} + \begin{bmatrix}(\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta))U_1\\ (\cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi))U_1\\ \cos(\phi)\cos(\theta)\,U_1\end{bmatrix} \quad (1.44)$$

Rewriting in term of the state variables **X, Y** and **Z.**

$$\ddot{x} = \frac{U_1}{m}(\sin(x_1)\sin(x_5) + \cos(x_1)\cos(x_5)\sin(x_3)) \quad (1.45)$$

$$\ddot{y} = \frac{U_1}{m}(\cos(x_1)\sin(x_5)\sin(x_3) - \cos(x_5)\sin(x_1)) \quad (1.46)$$

$$\ddot{z} = -g + \frac{U_1}{m}(\cos(x_1)\cos(x_3)) \quad (1.47)$$

The state equations present a mathematical model to study the drone behavior; in the next section control methods are discussed.

Chapter 02

PID Control of Quad-rotor

After developing a model for the quad-rotor, this model is then used to test the methods introduced in the previous chapter. In this chapter model parameters are listed, then the necessary demonstrations for the controllers are presented.

In this thesis three control methods were used: **PID**, Backstepping and Gain-scheduling **PID**.

2.1 Choice of Model Parameters

In this thesis the aircraft was modeled as a spherical center and four rectangular parallelepiped arms with the rotors being modeled as four spheres.

The physical parameters of the Quad-copter were as follows:

Parameter	**Symbol**	**Value**	**Unit**
Total mass	M (*)	0.8	kg
Spherical center mass	m_1	0.5	kg
Arm mass	m_2	0.04	kg
Rotor mass	m_3	0.05	kg
Spherical center radius	R	0.06	m
Arm dimensions (length × square sides)	l×a×a	0.1×0.03×0.03	m
Rotor radius	r	0.03	m
Inertia about **x-axis**	I_{xx}	0.0047	$Kg.m^2$
Inertia about **y-axis**	I_{yy}	0.0047	$Kg.m^2$
Inertia about **z-axis**	I_{zz}	0.0088	$Kg.m^2$
Rotor constant	K_f	1.8	-
Rotor constant	K_M	0.325	-

(*) $M= m_1+4\times m_2+4\times m_3$

Table 2.1: The physical parameters of the quad-copter.

2.2 PID Method

PID stands for proportional integral derivative controller, and it works by minimizing the error between desired and actual state. the proportional term depends only on the current error and changes the output proportionally to the error, whereas the integral term sums past values of the error and the derivative term is proportional to the rate of change of the error. Overall, the **PID** is used to stabilize linear plants and due to its ease of implementation it has a wide range of application.

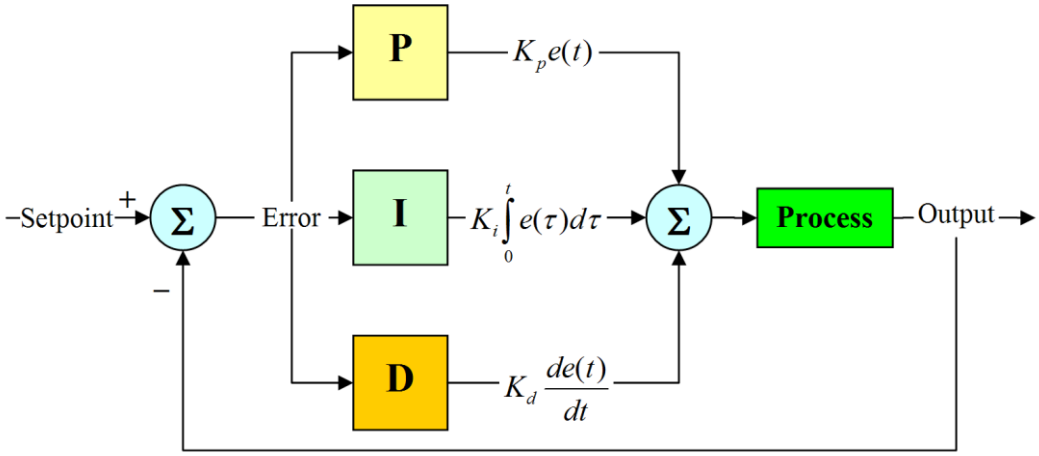


Fig2.1: PID controller

2.3 PID Parameter Tuning

After choosing the parameters for the model, SIMULINK tool was used to build the model and PID parameters were determined by multiple trials until satisfactory response was reached.

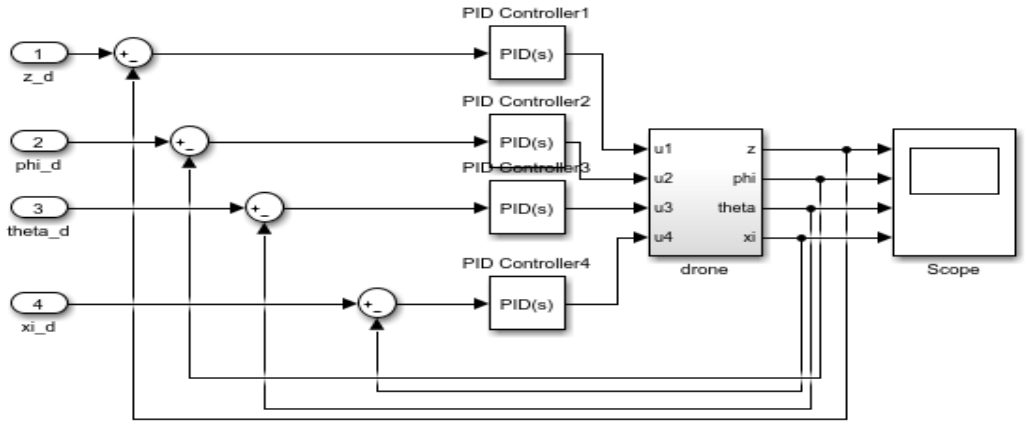


Fig. 2.2: PID controller tuning.

Z desired (m)		K	D	I	Steady state error	Percentage overshoot	Rise time(s)	Settling time(s)
Step (m)	1	20	3	6	0	0	0.5	6
	10	1	1	1	0.25	0.005	1.5	2.5
Ramp	10	1	20	5	0.4	-	-	-
Parabola	1	1	20	5	0	-	-	-
sine	1	2	20	20	0.5	-	-	

Table 2.2: PID parameters for altitude control.

Desired angle(deg)	K	D	I	Steady state error	Percentage overshoot
phi	1	60	60	0	0
theta	1	1	2	0	0
xi	1	1	1	0	0

Table 2.3: PID parameters for altitude control.

2.4 PID Method Results

Here PID controller is tested, first without disturbance and then with disturbance.

2.4.1 Response Without Disturbance

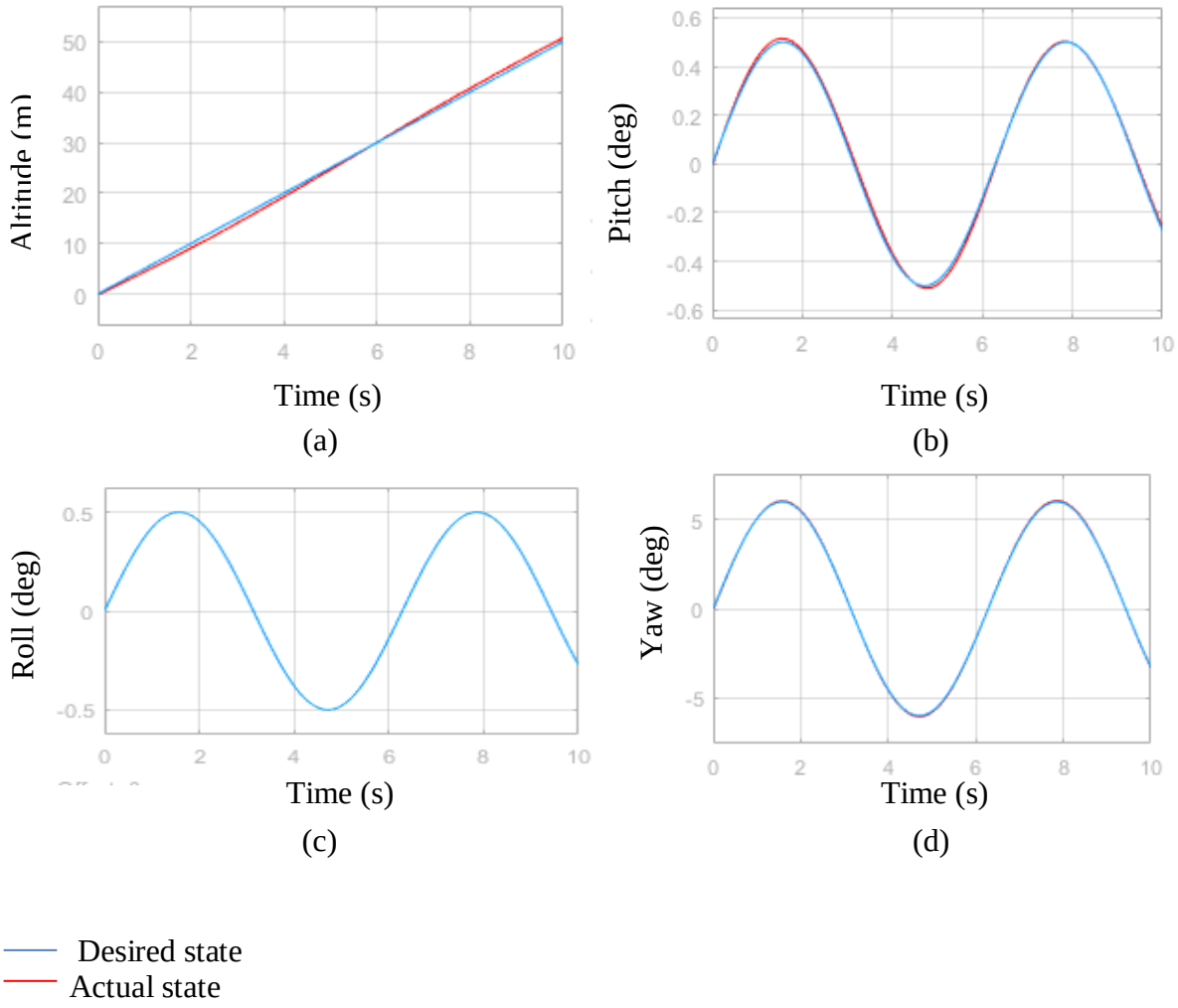


Fig. 2.3: PID response with no disturbance.

(a) Altitude response

(b) Pitch response

(c) roll response

(d) yaw response

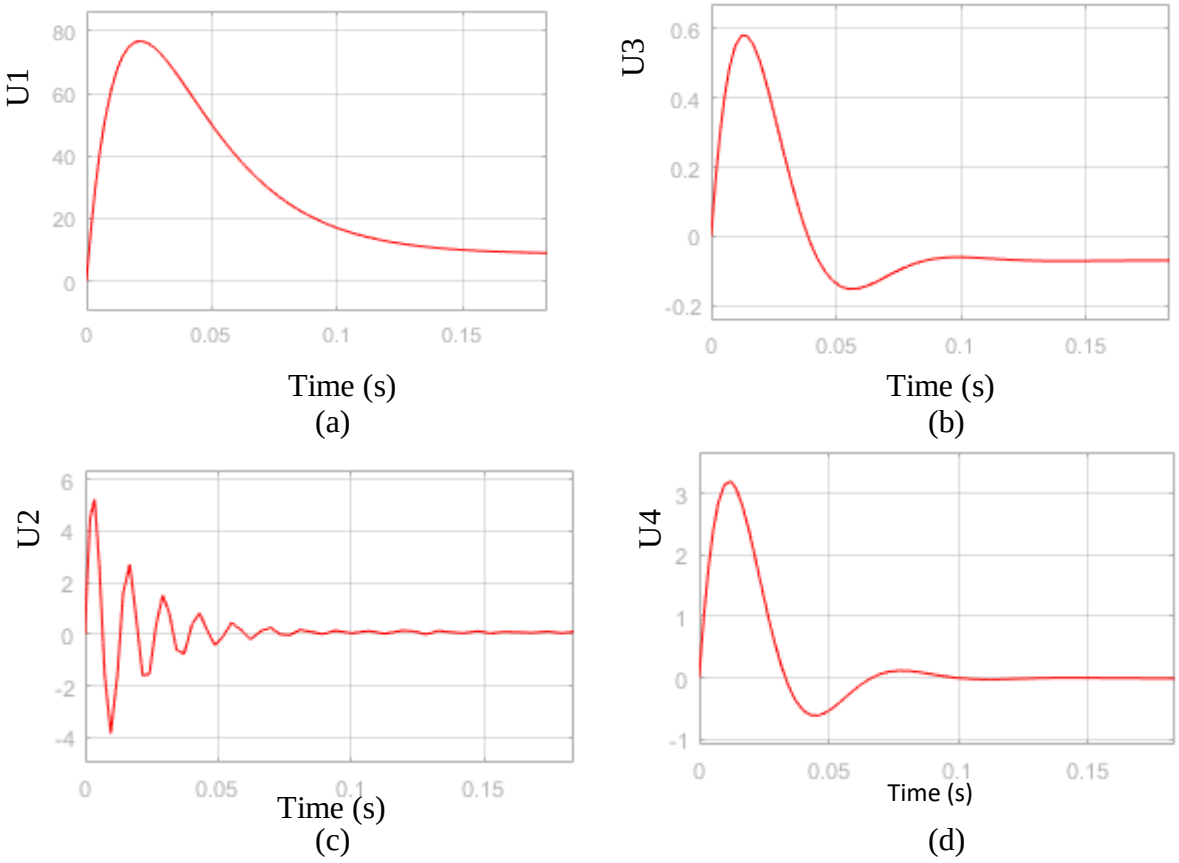


Fig. 2.4: Effective inputs without disturbance.

(a) U1

(b) U3

(c) U2

(d) U4

The resultant trajectory is:

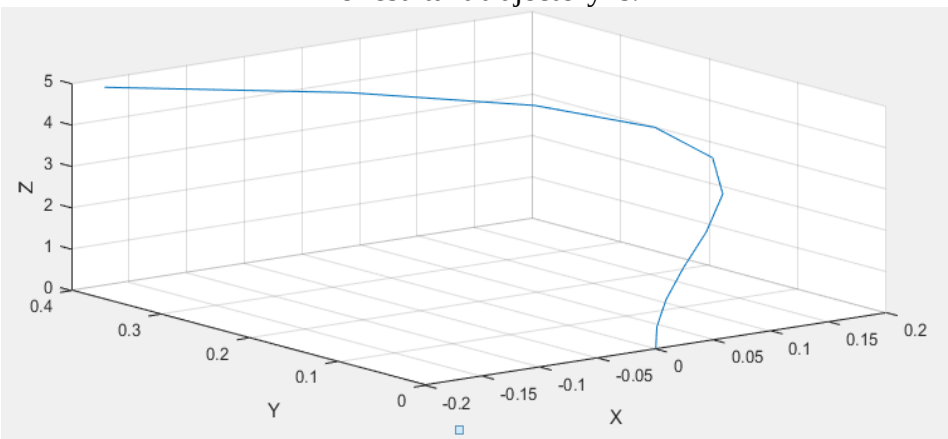


Fig. 2.5: Quad-copter trajectory.

2.4.2 Response With Disturbance

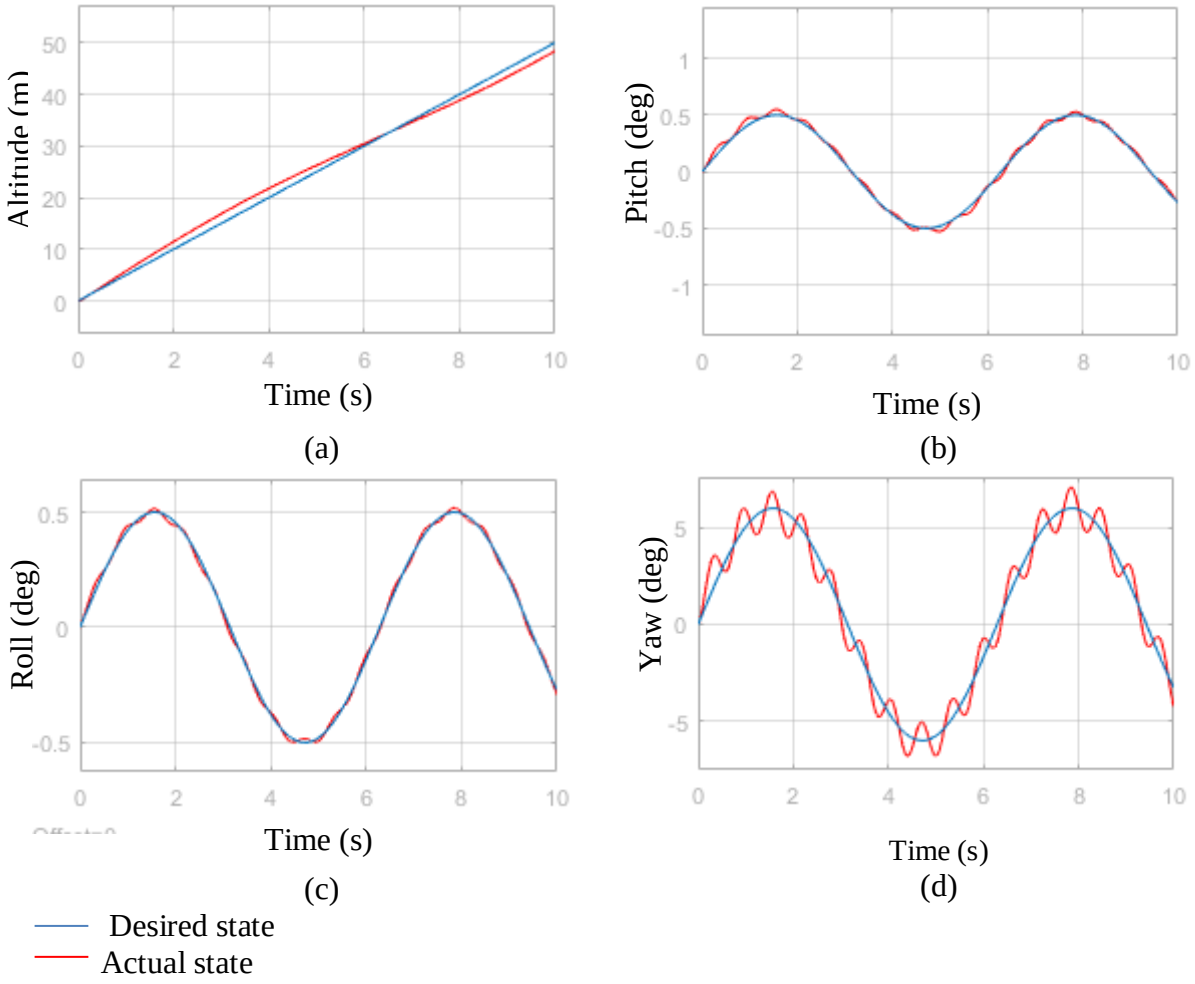


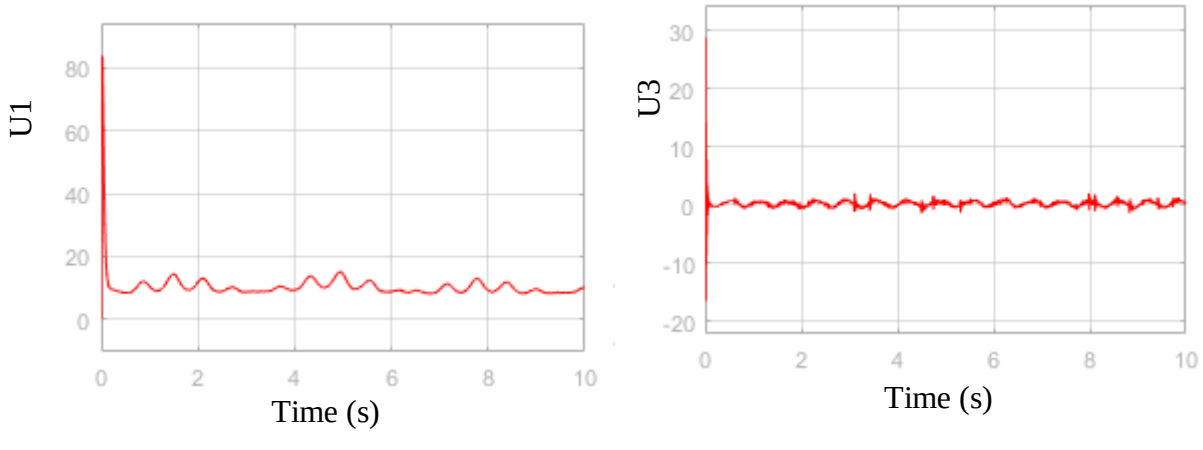
Fig. 2.6: PID response with disturbance.

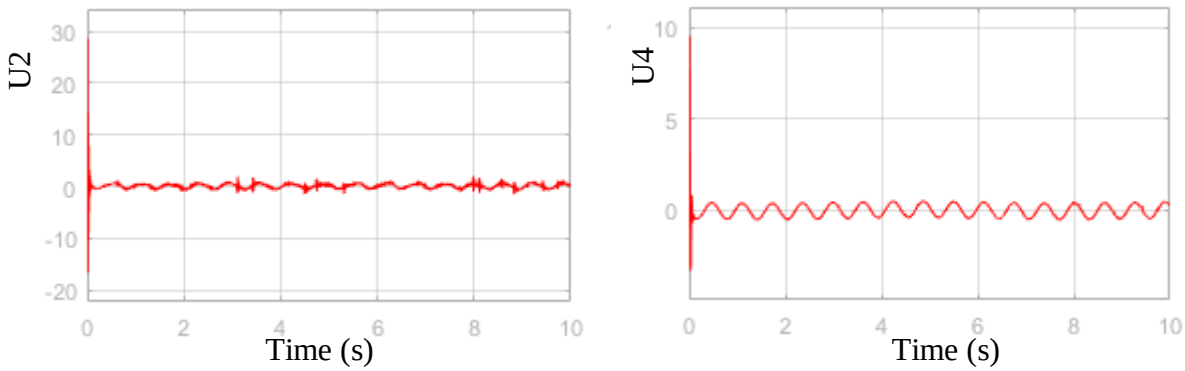
(a) Altitude response

(b) Pitch response

(c) roll response

(d) yaw response





(c) (d)

Fig. 2.7: Effective inputs with disturbance.

(a) U1 (b) U3

(c) U2 (d) U4

The **PID** is easy to use, understand and implement, the tuning process is simple and straight forward. However, it is not robust to disturbances and parameter variations. Moreover, the **PID** is a linear controller and is not suited to control a nonlinear plant since it doesn't produce a satisfactory output over the entire operation region.

Chapter 03

Backstepping Control of Quad-rotor

This is a technique that is used to control nonlinear systems, where the system is divided into a set of subsystems the number of which cannot exceed the degree of the system. Then, the control input is derived using Lyapunov stability principle. Because of the recursive nature of the Backstepping method, subsystems and progressively outer systems are stabilized, the process ends when the final outer system is stabilized.

The overall system is divided into two subsystems translational and rotational subsystems, the work presented in this section follows the method demonstrated in [5].

3.1 Roll Controller

Extracting the two first states of the state space model produces:

$$\dot{x_1} = x_2 \tag{3.1}$$

$$\dot{x_2} = x_4 x_6 a_1 - x_4 \Omega_r a_2 + b_1 U_2 \tag{3.2}$$

Lyapunov function is chosen as:

$$V_1 = \frac{1}{2} {z_1}^2 \tag{3.3}$$

Where z_1 is the error between the desired and actual roll angle .

$$z_1 = x_{1d} - x_1 \tag{3.4}$$

The time derivative of the Lyapunov function defined in Equation (3.3) is derived to be.

$$\dot{V_1} = z_1 \dot{z_1} \tag{3.5}$$

$$V_1 = z_1 \, (\dot{x_{1d}} - \dot{x_1}) \tag{3.6}$$

And from Equation (3.1) this can be written as,

$$\dot{V_1} = z_1 (\dot{x_{1d}} - x_2) \tag{3.7}$$

the system is stable system if the time derivative of a positive definite Lyapunov function is negative semi definite. So the function $W_1(z) = -c_1 {z_1}^2$is chosen to bound $\dot{V_1}$.

$$\dot{V_1} = z_1 (\dot{x_{1d}} - x_2) \leq -c_1 {z_1}^2 \tag{3.8}$$

Where c_1 is a positive constant. To satisfy this inequality the virtual control input can be chosen to be,

$$(x_2)_{desired} = \dot{x_{1d}} + c_1 z_1 \tag{3.9}$$

Definning a new error variable z_2 to be the deviation of the state x_2 from its desired value,

$$z_2 = x_2 - \dot{x_{1d}} - c_1 z_1 \tag{3.10}$$

Substituting by the value of x_2 from the last equation in equation (3.7) we get,

$$\dot{V_1} = z_1 \dot{z_1}$$

$$V_1 = z_1 (\dot{x_{1d}} - x_2)$$

$$V_1 = z_1 (\dot{x_{1d}} - (z_2 + \dot{x_{1d}} + c_1 z_1))$$

$$V_1 = -z_1 z_2 - c_1 {z_1}^2 \tag{3.11}$$

The next step is to augment the first Lyapunov function V_1 with a quadratic term in the second error variable z_2 to get a positive definite V_2 :

$$V_2 = V_1 + \frac{1}{2}{z_2}^2 \tag{3.12}$$

With time derivative,

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2$$

$$\dot{V}_2 = -z_1z_2 - c_1{z_1}^2 + z_2(\dot{x_2} - \ddot{x_{1d}} - c_1\dot{z_1}) \tag{3.13}$$

Choosing $W_2(\mathrm{z}) = -c_1{z_1}^2 - c_2{z_2}^2$to be the positive definite bounding function, with c_2 is positive constant and by replacing the value of $\dot{x_2}$ from equation (3.2) leads to the following inequality,

$$\dot{V}_2 = -z_1z_2 - c_1{z_1}^2 + z_2(x_4x_6a_1 - x_4\Omega_ra_2 + b_1U_2 - \ddot{x_{1d}} - c_1\dot{z_1})$$
$$\leq -c_1{z_1}^2 - c_2{z_2}^2 \tag{3.14}$$

Simplifying the last inequality, the control input U_2 can be written as,

$$U_2 = \frac{1}{b_1}(-c_2z_2 + z_1 - x_4x_6a_1 + x_4\Omega_ra_2 + \ddot{x_{1d}} + c_1\dot{x_{1d}} - c_1x_2) \tag{3.15}$$

3.2 Pitch Controller

To derive the pitch controller we follow the same steps as we did for the Roll controller. Extracting the pitch angle and its rate of change from the state space model in Equation (1.39):

$$\dot{x_3} = x_4 \quad (3.16)$$

$$\dot{x_4} = x_2 x_6 a_3 + x_2 \Omega_r a_4 + b_2 U_3 \quad (3.17)$$

And the error in pitch is defined as $z_3 = x_{3d} - x_3$ leading to a positive definite lyapunov function,

$$V_3 = \frac{1}{2} {z_3}^2 \quad (3.18)$$

With time derivative,

$$\dot{V_3} = z_3 \dot{z_3}$$

$$\dot{V_3} = z_3(\dot{x_{3d}} - x_4) \quad (3.19)$$

Choosing $W_3(z) = -c_3 {z_3}^2$ to bethe bounding function where c_3 a positive constant, the desired x_4 state is:

$$(x_4)_{desired} = \dot{x_{3d}} + c_3 z_3 \quad (3.20)$$

And the error in state x_4 is,

$$z_4 = x_4 - \dot{x_{3d}} - c_3 z_3 \quad (3.21)$$

Substituting by the value of x_4 from the last equation in equation (3.19) we get:

$\dot{V_3} = z_3 \dot{z_3}$

$\dot{V_3} = z_3\left(\dot{x_{3d}} - (z_4 + \dot{x_{3d}} + c_3 z_3)\right)$

$$\dot{V_3} = -z_3 z_4 - c_3 {z_3}^2 \quad (3.22)$$

Augmenting the previous Lyapunov function we get:

$$V_4 = V_3 + \frac{1}{2} {Z_4}^2 \quad (3.23)$$

Defining a new bounding function to be $W_4(z) = -c_3 {z_3}^2 - c_4 {z_4}^2$ with:

- c_4 a positive constant:

$$\dot{V_4} = -z_3 z_4 - c_3 {z_3}^2 + z_4(\dot{x_4} - \ddot{x_{3d}} - c_3 z_3) \leq -c_3 {z_3}^2 - c_4 {z_4}^2 \quad (3.24)$$

Replacing x_4 with its definition from equation (3.17) and solving for U_3.The roll angle control input is found to be,

$$U_3 = \frac{1}{b_2}(-c_4 z_4 + z_3 - x_2 x_6 a_3 - x_2 \Omega_r a_4 + \ddot{x_{3d}} + c_3 \dot{x_{3d}} - c_3 x_4) \quad (3.25)$$

3.3 Yaw Controller

Following exactly the same steps as the roll and pitch controllers, the control input for the yaw angle is derived to be,

$$U_4 = \frac{1}{b_3}(-c_6 z_6 + z_5 - x_2 x_4 a_5 + \ddot{x}_{5d} + c_5 \dot{x}_{5d} - c_5 x_6) \quad (3.26)$$

With :

$$z_5 = x_{5d} - x_5 \quad (3.27)$$

$$z_6 = x_6 - \dot{x}_{5d} - c_5 z_5 \quad (3.28)$$

- And c_5 and c_6 are positive constants.

3.4 Altitude Controller

For the altitude controller, the control input U_1 is derived in the same manner as $U_2, U_3, and U_4$ to be

$$U_1 = \frac{m}{\cos x_1 \cos x_3}(z_7 + g - \ddot{x}_{7d} - c_7 \dot{x}_{7d} + c_7 x_8 - c_8 z_8) \quad (3.29)$$

With :

$$z_7 = x_{7d} - x_7 \quad (3.30)$$

$$z_8 = x_8 - \dot{x}_{7d} - c_7 z_7 \quad (3.31)$$

And c_7 and c_8 are positive constants.

The stabilizing constants were chosen as follows:

- c1 = 0.01
- c2 = 28
- c3 = 0.01
- c4 = 28
- c5 = 0.1
- c6 = 25
- c7 = 2
- c8 = 30

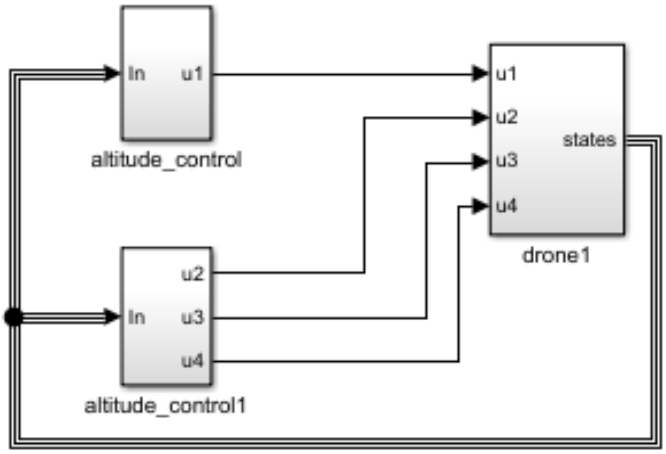


Fig. 3.1: Backstepping controller.

3.5 Backstepping Method Results

3.5.1 Response without Disturbance

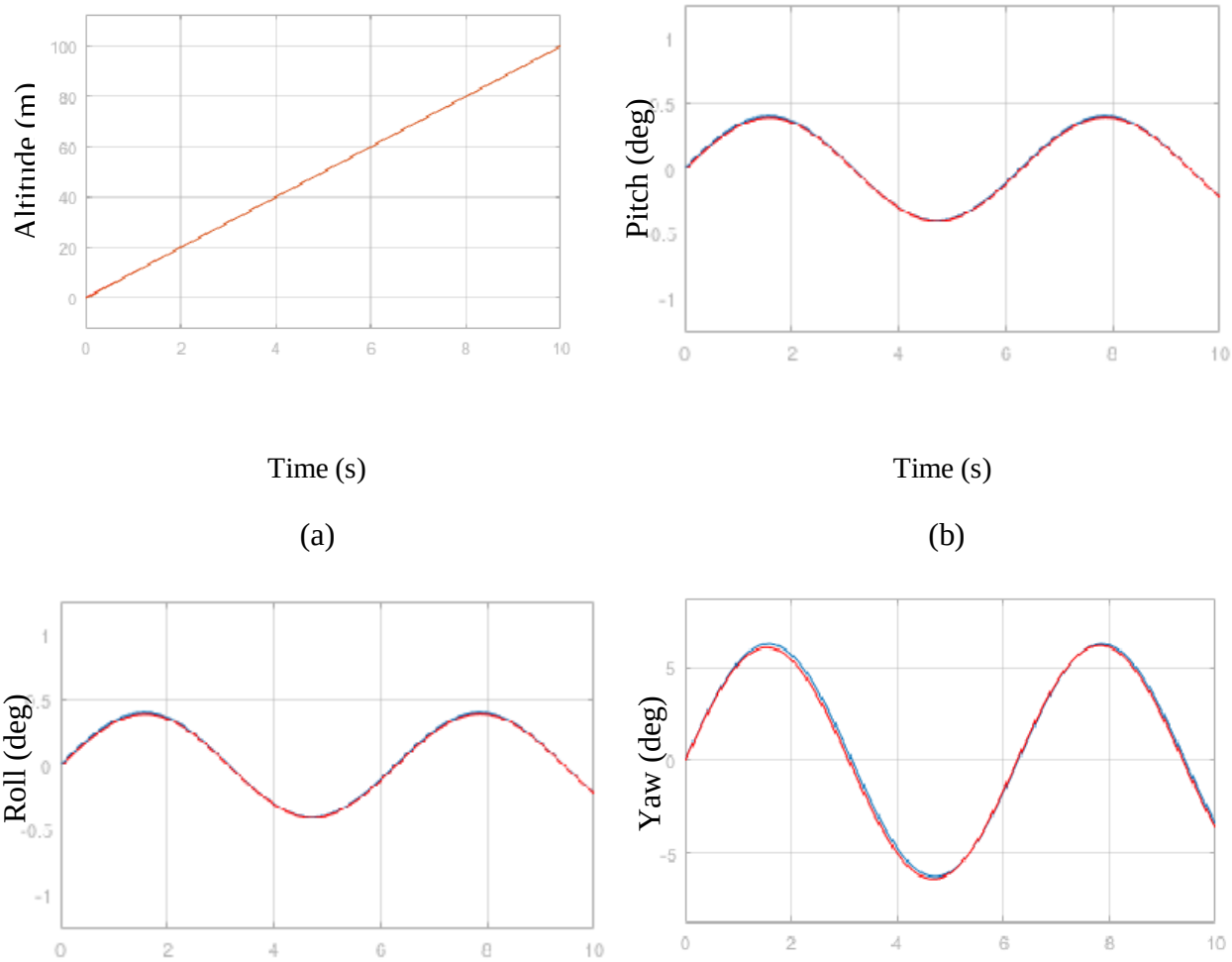


Fig. 3.2: Backstepping response with no disturbance.

(a) Altitude response (b) Pitch response

(c) roll response (d) yaw response

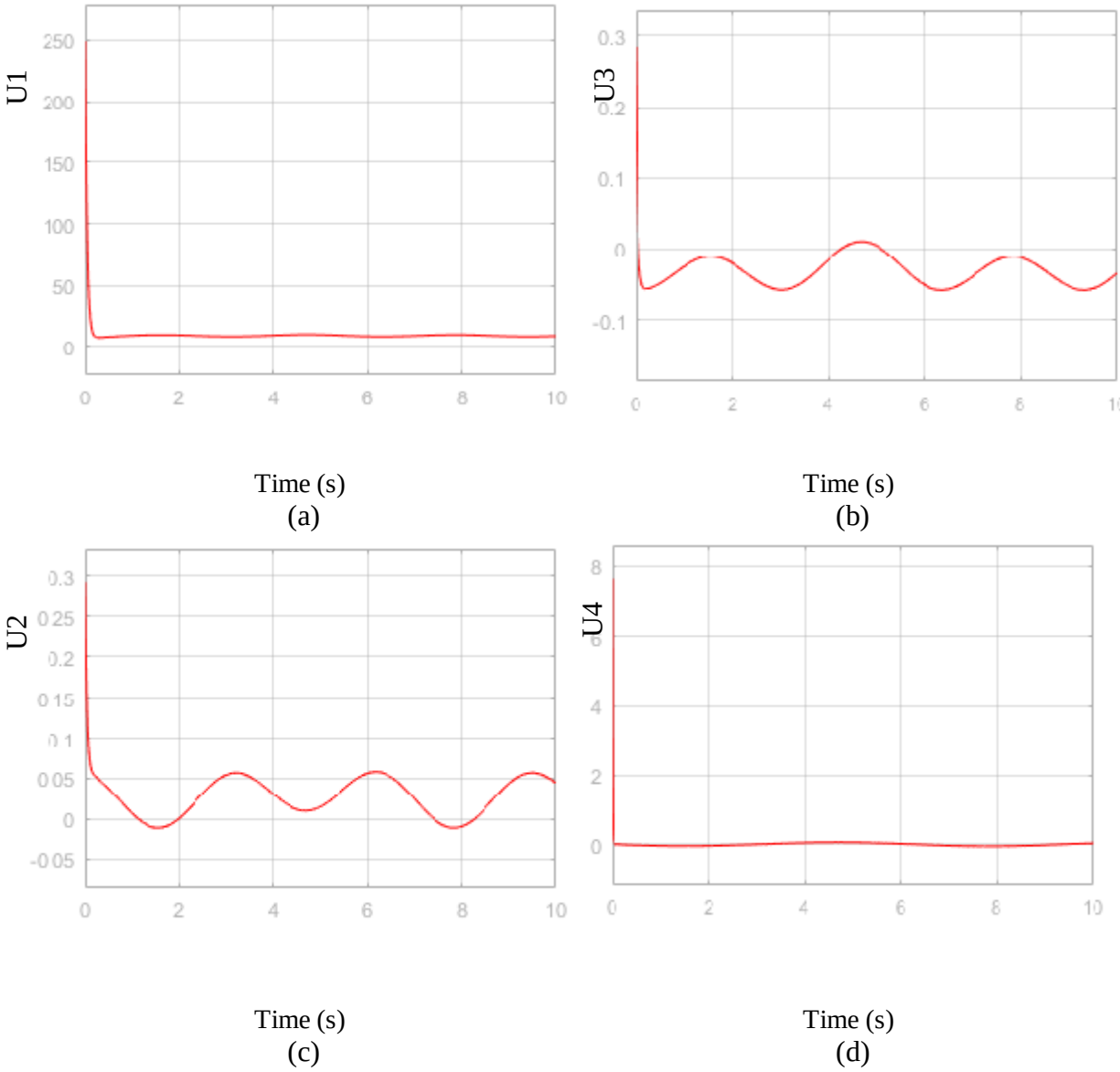


Fig. 3.3: Effective inputs without disturbance.

(a) U1 (b) U3

(c) U2 (d) U4

3.5.2 Response with Disturbance

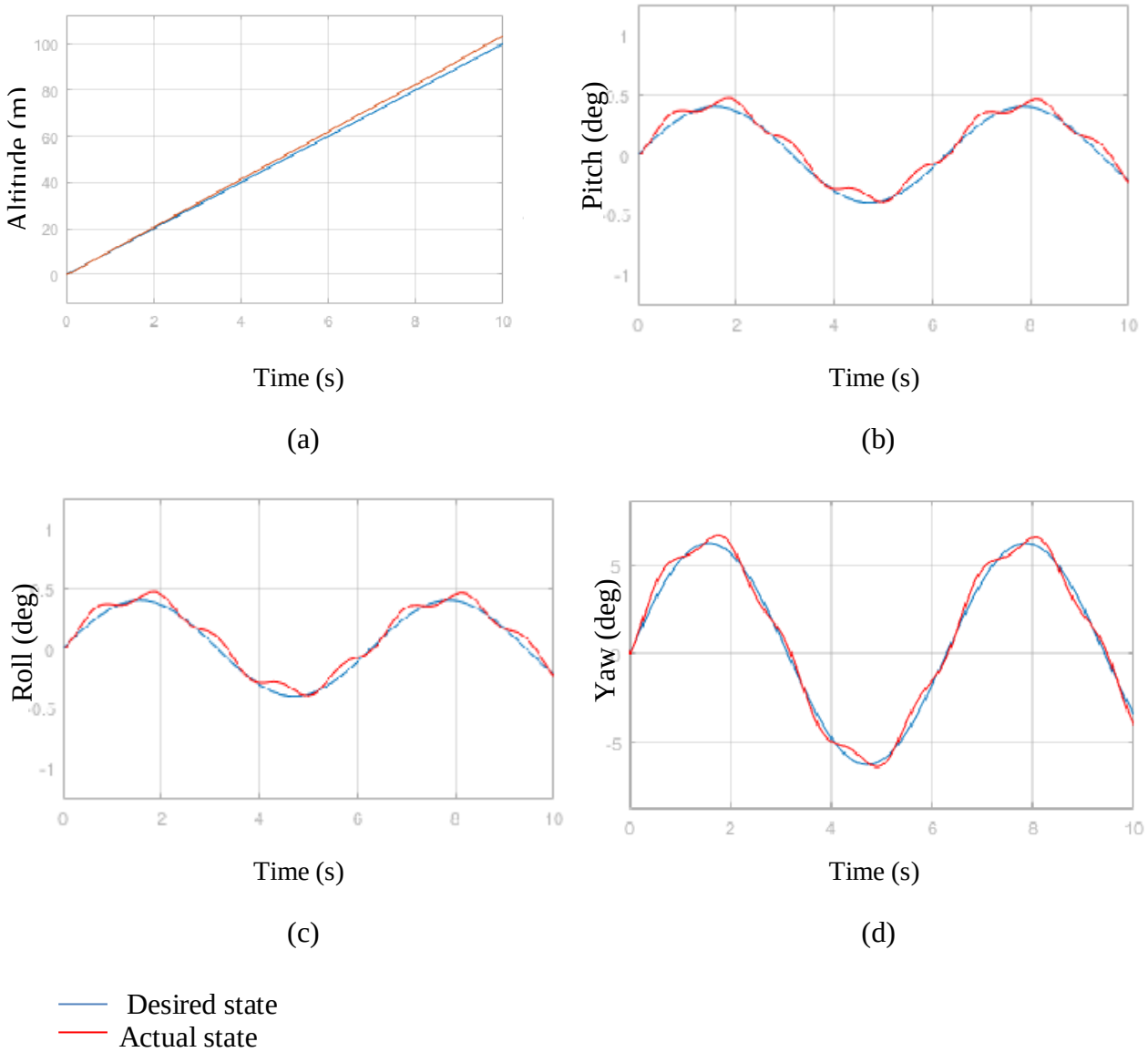
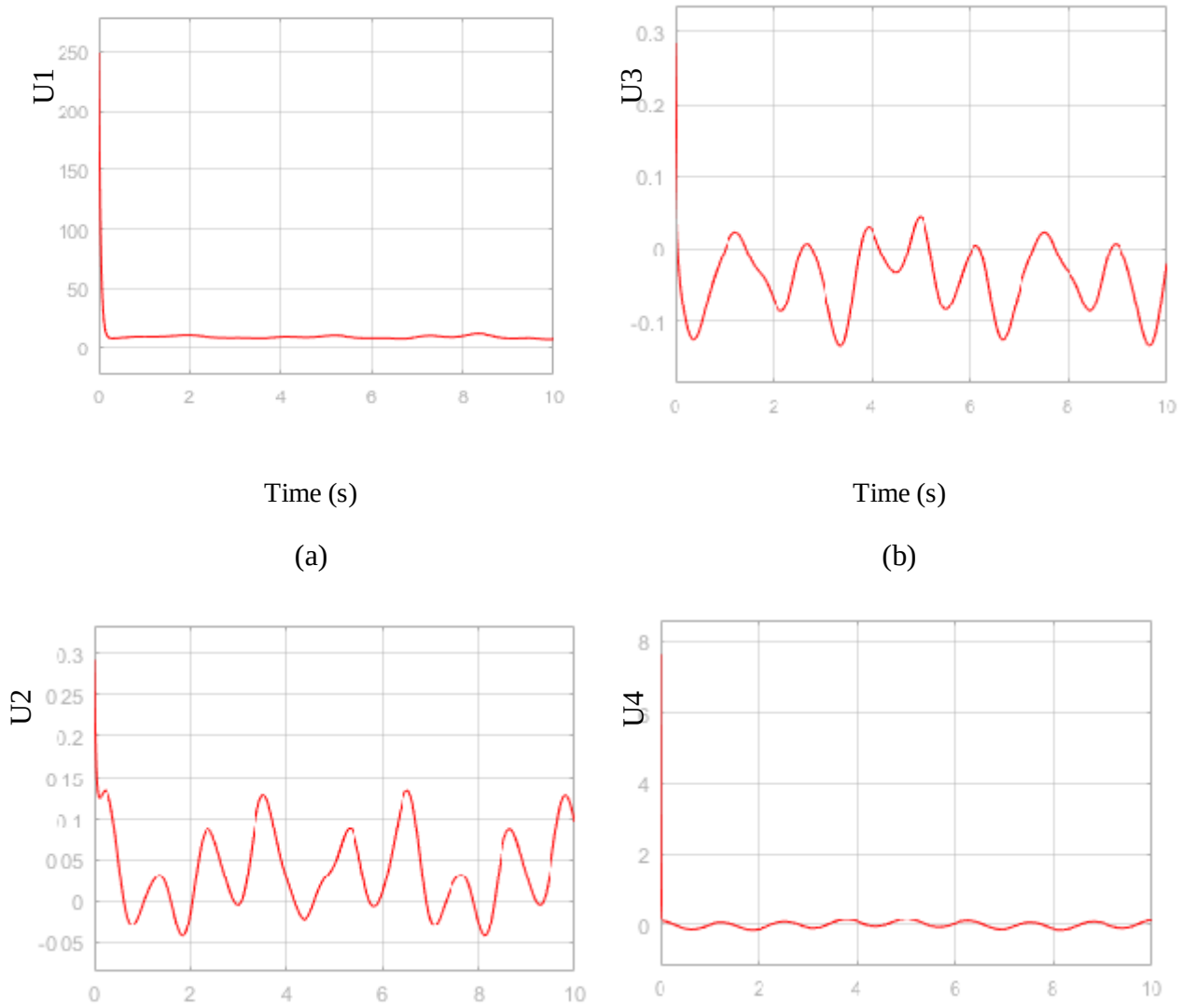


Fig 3.4: Backstepping response with disturbance.

(a) Altitude response (b) Pitch response

(c) roll response (d) yaw response



(a) (b)

(c) (d)

Fig 3.5: Effective inputs with disturbance.

(b) U1 (b) U3

(c) U2 (d) U4

The second method was Backstepping and was specifically designed for a class of nonlinear systems so it does not imply plant linearization and in ensures Lyapunov stability, on the other hand this method requires tuning several parameters and getting the control inputs is mathematically tasking.

Chapter 04

Gain-scheduling Control of Quad-rotor

4.1 Gain-scheduling Method

Because of the nonlinear nature of the plant, Gain-scheduling method uses a linear controller for each operating point of the system, thus the linear controller provides satisfactory control over a range of values of the scheduling variable which determines the appropriate controller to be used.

Multiple PIDs are used so that each one gives satisfactory control in a different operating point, choosing the desired states as scheduling variables.

Starting with altitude control, PID parameter values were as follows:

Z desired (m)	K	D	I	Steady state error	Percentage overshoot	Rise time(s)	Settling time(s)
1	20	3	0	0.2	0	0.25	-
2	7	2.3	0	0.5	0	0.5	-
5	3	1	1	0	0	1	11.9
10	2	1	0	0.4	0	1	1.6
20	0.9	0	0	0	0	2.25	2.4

Table 4.1: Gain-scheduling PID parameters for altitude control.

In the same manner the other PID are tuned to be:

phi desired (deg)	K	D	I	Steady state error	Percentage overshoot	Rise time(s)	Settling time(s)
0.1	0.1	0.1	0	0.02	0.1	5	-
11	0.1	0.1	0	0.1	0	5	12
12	0.05	0.06	0.01	0	0.3	3.3	18
13	0.01	0.03	0	0.9	0	9	12
20	0.1	0.1	0	0	0	5.1	5.7

Table 4.2: Gain-scheduling PID parameters for roll control.

theta desired (deg)	K	D	I	Steady state error	Percentage overshoot	Rise time(s)	Settling time(s)
11	0.1	0.1	0	0	1.7	0.3	3.5
12	0.05	0.05	0	0	1	0.2	6.2
14	0.05	0.05	0	0	0.7	0.25	6
18	0.06	0.06	0.01	0	0.6	0.3	12.5
25	0.06	0.06	0.01	0	0.7	1.5	30

Table 4.3: Gain-scheduling PID parameters for pitch control.

xi desired (deg)	K	D	I	Steady state error	Percentage overshoot	Rise time(s)	Settling time(s)
1	0.5	0.025	0	0.1	0	0.16	0.2
2	0.5	0.01	0.05	0.8	0.1	0.25	-
3	0.1	0.02	0.01	2	0	0.4	-
9	0.09	0.03	0	1.5	0.7	0.5	-
20	0.09	0.03	0	1.9	0	0.75	-

Table 4.4: Gain-scheduling PID parameters for yaw control.

The controllers discussed here are used in the next chapter where the results are presented.

4.2 Gain-scheduling Method Results

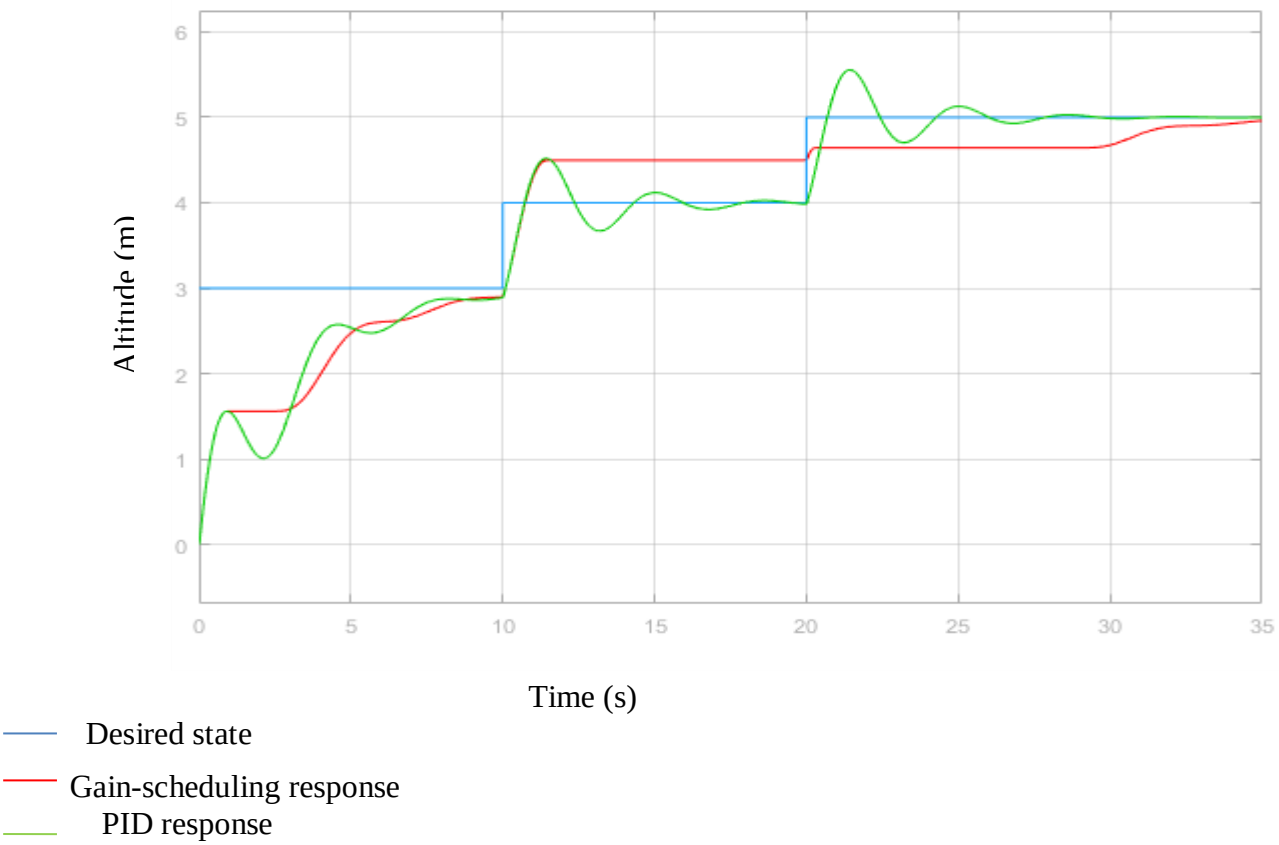


Fig 4.1: Altitude response.

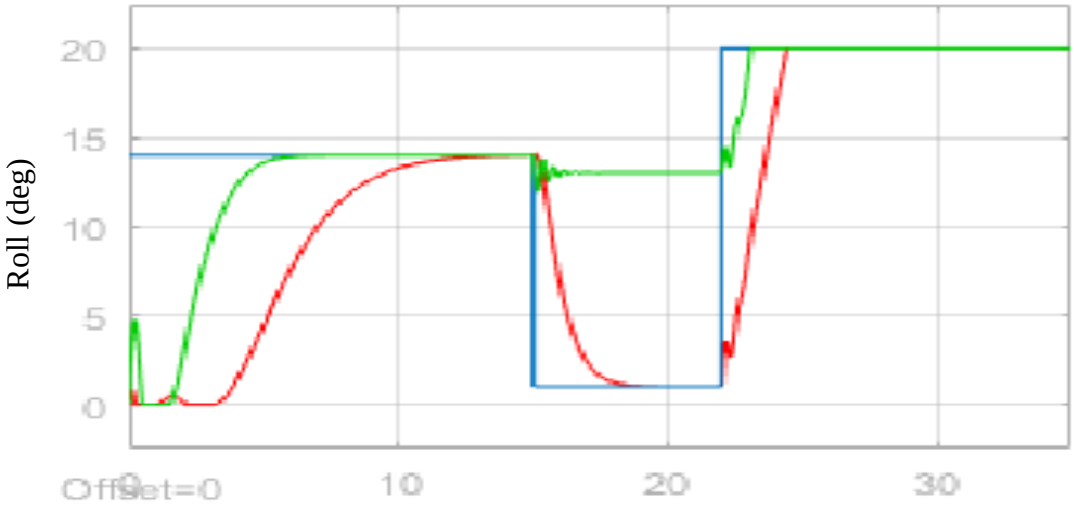


Fig 4.2: Roll response.

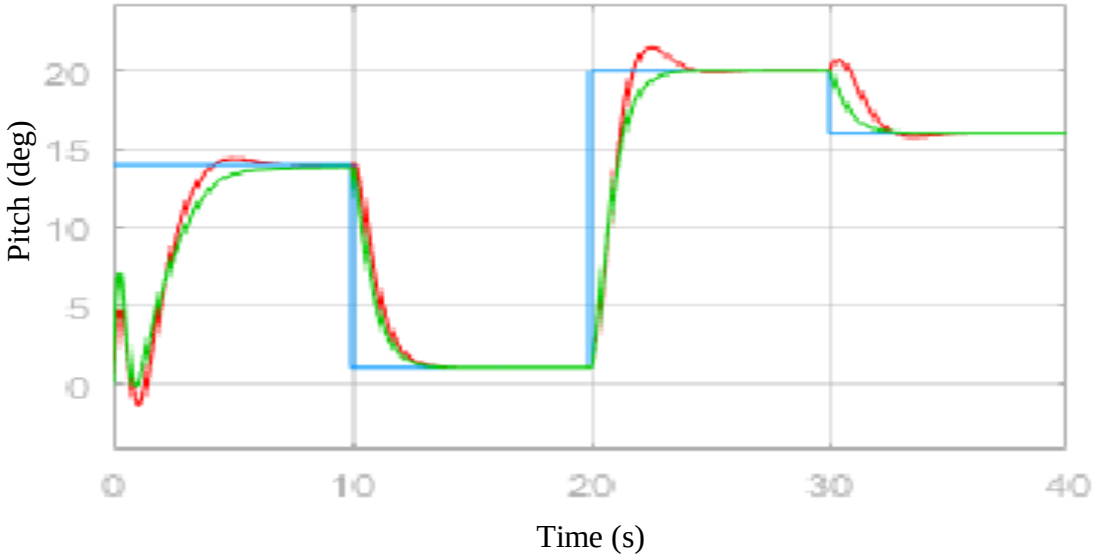


Fig 4.3: Pitch response.

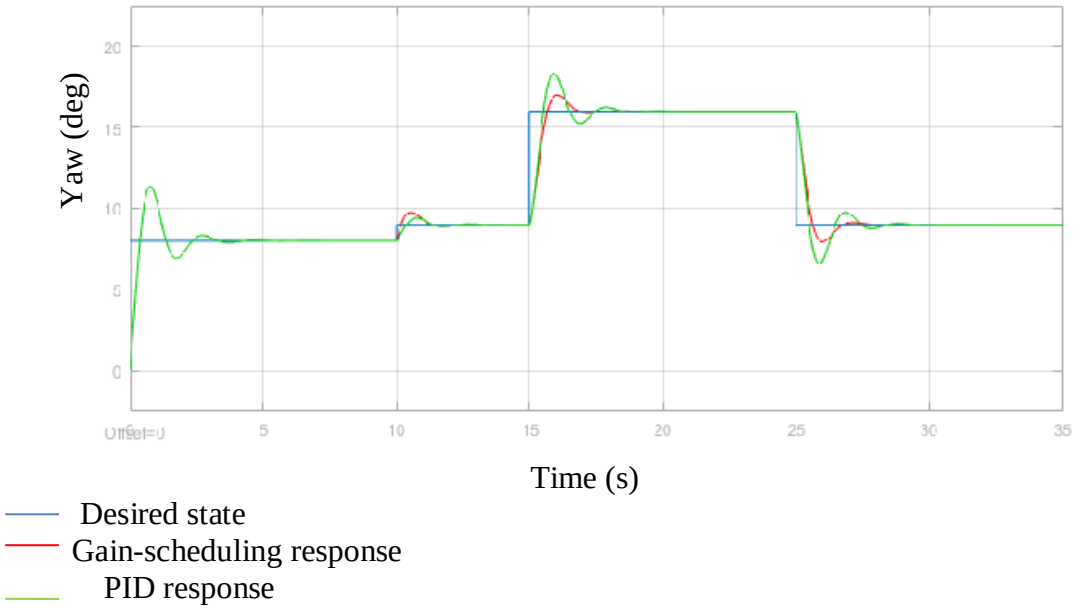


Fig 4.4: Pitch response.

Gain-scheduling methods is based on multiple controllers enabling it to offer satisfactory behavior in several operation points, the down side to that is tuning multiple PIDs. Also, the control input had spikes when switching between controllers.

4.3 Comparison between the Three Developed Controllers

In the region near equilibrium the three controllers give similar results, however away from equilibrium **PID** was unable to control the Quad-copter unlike Backsttepping and Gain-scheduling controllers, but Gain-scheduling controller suffered from spikes in the control input at switching instances.

Conclusion

In summary, in this thesis a model for the Quad-copter was developed after studying the nature of **6DOF** motion and the needed rotations, as well as the various forces and moments acting on the quad-copter. From that, state space equations were used to describe the behavior of the system.

This was then used to build equivalent system in Simulink, the quad-copter was assumed to be a rigid body with rigid propellers, it was modeled as a sphere with arms, the aircraft was assumed to be symmetric with body axes passing through its axes of symmetry.

Three methods of control were tested on the system. These methods are: **PID**, Backstepping and Gain-scheduling. They were simulated in MATLAB/Simulink environment, The results acquired were that there is no difference between the controllers in the linear region, the linear nature of **PID** prevents it from being a suitable method away of the equilibrium, Gain-scheduling **PID** provided batter control than **PID** controller and Backstepping method was suitable for the quad-copter given that it is a nonlinear control method. A drawback of the Backstepping method is the mathematical derivation. As for Gain-scheduling **PID** tuning multiple controllers can be time consuming, in addition to that spikes are present in the control inputs.

To conclude the Backstepping method was the most suitable for the system because the complexity of input derivation is compensated by the fact that it provides good control for any operation point and has fewer tuning parameter compared to Gain-scheduling **PID.**

Future work

In future work, it would be useful to robust the **PID** controller. In addition to that, other functionality could be added to the quad-copter like obstacle detection and avoidance, or automating the quad-copter to follow a predefined path in applications where such function is needed.

References

[1]E. Gopalakrishnan," QUAD-COPTER FLIGHT MECHANICS MODEL AND CONTROL ALGORITHMS", Czech Technical University, Prague, May 2016.

[2]A. Gharatapeh, "Modelling and control of Quad-copter School of Science Mat-2.4108 Independent research project in applied mathematics A", *Academia.edu*, 2021. [Online]. Available: https://www.academia.edu/33664128/Modelling_and_control_of_Quad-copter_School_of_Science_Mat_2_4108_Independent_research_project_in_applied_mathematics_A// . [Accessed: 06- Jul- 2021].

[3]"Demystifying Drone Dynamics!", *Towards Data Science*, 2018. [Online]. Available: https://towardsdatascience.com/demystifying-drone-dynamics-ee98b1ba882f// . [Accessed: 06- Jul- 2021].

[4]N. Hall, "Beginner's Guide to Aerodynamics", *NATIONAL AERNAUTICS AND SPACE ADMINISTRATION*, 2021. [Online]. Available: https://www.grc.nasa.gov/www/k-12/airplane/bga.html// . [Accessed: 06- Jul- 2021].

[4]Ang, K.H. and Chong, G.C.Y. and Li, Y. (2005) PID control system analysis, design, and technology. IEEE Transactions on Control Systems Technology 13(4):pp. 559-576. http://eprints.gla.ac.uk/3817/

[5]. S. Bouabdallah and R. Siegwart. Backstepping and sliding-mode techniques applied to an indoor micro Quad-rotor. In Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on, pages 2247– 2252, 2005. doi: 10.1109/ROBOT.2005.1570447.

[6]D. J.Leith, "Survey of gain-scheduling analysis and design", *Taylor & Francis*, 2021. [Online]. Available: http://mural.maynoothuniversity.ie/1834/1/1001965764_link_19992.pdf// . [Accessed: 06- Jul- 2021].

[7]W. R.Young, The helicopters. Alexandria, Va. : Time-Life Books, 1982.

[8]R. Jazar, *Advanced Dynamics: Rigid Body, Multibody, and Aerospace Applications*, 1st ed. Wiley, 2011.

[9]A. Pytel and J. Kiusalaas, *Engineering mechanics*. Independence, Ky.: Nelson Engineering, 2009.

[10]A. Hystad, "Model, Design and Control of a Quadcopter", 2015. [Accessed 17 July 2021].

[11]T. Jiinec, "Stabilization and Control of Unmanned Quadcopter", 2011. [Accessed 17 July 2021].

[12]N. Krasnov, *Aerodynamics: Fundamentals of Theory, Aerodynamics of an Airfoil and Wing; Methods of Aerodynamic*. 1986.

[13]F. Hitchens, *Propeller Aerodynamics*. Luton, Bedfordshire: Andrews UK, 2015.

[14]E. Houghton, P. Carpenter, S. Collicott and D. Valentine, *Aerodynamics for engineering students*. Amsterdam: Butterworth-Heinemann, 2017.

[15]T. Luukkonen, "Modelling and control of quadcopter", 2011. [Accessed 19 July 2021].

[16]E. Kuantama, T. Vesselenyi, S. Dzitac and R. Tarca, "PID and Fuzzy-PID Control Model for Quadcopter Attitude with Disturbance Parameter", *International Journal of*

Computers Communications & Control, vol. 12, no. 4, p. 519, 2017. Available: 10.15837/ijccc.2017.4.2962.
[17]V. Praveen and A. PillaI, 2016. [Accessed 19 July 2021].
[18]U. Fiaz, A. Mukarram and U. Khan, "Altitude Control of a Quadcopter", 2015. [Accessed 19 July 2021].
[19]G. Ostojic, S. Stankovski, B. Tejic and N. Đukić, "Design, Control and Application of Quadcopter", 2015. [Accessed 19 July 2021].
[20]. Miroslav Krstic, Petar V Kokotovic, and IoannisKanellakopoulos. *Nonlinearand adaptive control design*. John Wiley & Sons, Inc., 1995.