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Title

Multi variable stabilization control of a hydraulic system

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Dedication

This work is lovingly dedicated to : To the unwavering pillars of love and support, my beloved mother and father, whose boundless affection and nurturing embrace have been an unwavering constant throughout my life's journey, providing a profound sense of security and comfort that has never left me to face the world alone.

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Summary

list of figures

Nomenclature

LTI	Linear Time Invariant
TS	Takagi-Sugeno
CS	Compressed Sensing
LMI	Linear Matrix Inequality
GAS	Globally Asymptotically Sable
PDC	Pparallel Distributed compensation
GEVP	Genral Eigenvalu Problem

GENERAL INTRODUCTION

General introduction

Many real-world physical dynamical systems exhibit complex nonlinear behavior that cannot be adequately captured by traditional linear modeling approaches. While linear control techniques relying on linearization around operating points can work for small ranges, they become ineffective and potentially unstable when dealing with large operating ranges where nonlinearities play a significant role. This limitation is especially pronounced in applications like hydraulic and mechanical systems with inherent nonlinearities. Furthermore, precise mathematical modeling of such systems is often impractical due to uncertainties in parameters and lack of full input-output data.
To address these challenges, this thesis explores the application of the Takagi-Sugeno (TS) fuzzy modeling approach for accurately representing nonlinear hydraulic systems. The TS fuzzy model employs a multi-model fuzzy architecture composed of multiple linear sub-models that locally capture the system dynamics, combined through fuzzy rules to represent the overall nonlinear behavior. Where this thesis is structured as follows :

Chapter 1 establishes the fundamental theoretical background by introducing key concepts in automatic control systems such as linearity, non-linearity, static and dynamic system analysis, as well as instrumentation principles for hydraulic systems. This lays the groundwork for subsequent discussions on nonlinear modeling and control.

Chapter 2 establishes the essential notations and theoretical foundations required for this work. Specifically, it introduces the dynamic Takagi-Sugeno fuzzy system formulation that will be employed throughout the thesis, the chapter also presents two key methods used for constructing TS fuzzy models from a given nonlinear system, the first approach, the sector nonlinearity method, enables deriving an exact TS fuzzy representation of the nonlinear system within a compact region of the state-space. Alternatively, the second technique utilizes Taylor series expansions around multiple operating points, these model construction methods lay the groundwork for the subsequent application of TS fuzzy modeling to the hydraulic system under consideration.

In Chapter 3, the TS fuzzy modeling approach is applied to develop an accurate multi-model representation of a complex nonlinear hydraulic three-tank system, using simulations to validate the TS fuzzy model against a conventional dynamic model, demonstrating its capability to precisely capture the system's behavior while providing a simpler modeling framework.

Chapter 4 delves into stability analysis and control design techniques for TS fuzzy models using linear matrix inequality (LMI) constraints and lyapunov's direct method. Stabilization results based on quadratic lyapunov functions are presented, showcasing the advantages of the TS fuzzy approach for control applications.

Overall, This thesis presents an innovative fuzzy modeling and control approach specifically designed for intricate nonlinear hydraulic systems, proving that TS fuzzy approach overcomes limitations of linear techniques while providing an intuitive multi-model architecture amenable to stability analysis and control design.

Chapter I

GENERALITIES

I.1 Introduction

In this chapter, we embark on a journey through the fundamental concepts that underpin the analysis of systems. We will explore the fundamental principles of automatic control systems, with a focus on the key concepts of linearity and non-linearity and key distinctions between the two. Dynamic and static systems emerge as essential categories within our discourse, with dynamic systems exhibiting evolving behaviors, contrasting with static systems characterized by unchanging states. Through dynamic system analysis, we unravel the intricate interplay of variables and dynamics that shape system behavior, paving the way for precise modeling.

Understanding these concepts is essential for effectively designing, modeling, and controlling various types of systems. Furthermore, this chapter will also cover the basics of instrumentation, particularly concentrating on the instrumentation of hydraulic systems, where we shine a spotlight on the sensors and actuators employed in hydraulic systems, showcasing their indispensable role in translating physical phenomena into measurable signals and actionable commands.

This chapter will lay a solid foundation for our deeper exploration of automatic control systems, providing us with the essential knowledge for a thorough understanding of the upcoming concepts and applications.

I.2 Preliminary definitions

Before we can discuss control systems, some basic terminologies must be defined.

- Controlled Variable and Control Signal or Manipulated Variable : The controlled variable is the quantity or condition that is measured and controlled.The control signal or manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system, control means measuring the value of the controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value. In studying control engineering, we need to define additional terms that are necessary to describe control systems.

- Plants : A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

- Disturbances : A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called internal, while an external disturbance is generated outside the system and taken as an input.

- Processes : A process is a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result, or an artificial or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or an end. In this book we shall call any operation to be controlled a process. Examples are chemical, economic, and biological processes.
- Feedback Control : Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

I.3 The concept of a system

In the field of automation and automatic control, a system refers to an interconnected set of components that work together to regulate, command, or control an operation or process without direct human intervention.

1. Physical Process/Plant : This is the entity being controlled, such as an industrial process, machine, vehicle, or other operational technology. It has inputs that cause its behavior.
2. Sensors : Devices that measure and provide signal feedback about the current state or output of the physical process.
3. Controllers/Control System : This computational component takes in sensor data as inputs, processes it through control algorithms or logic, and determines the necessary control actions.
4. Actuators : Devices that can influence or manipulate the physical process based on the commands from the controller.

The goal is to automatically maintain desired operating conditions or outputs from the physical process/plant by constantly measuring outputs, computing necessary corrective inputs using the control system, and implementing those inputs via actuators - all in a closed loop without human involvement.

External disturbances like noise, environmental changes, etc. can also influence the process, which the control system must compensate for.
So a control system is a combination or arrangement of a number of different physical components to form a whole unit such that combining unit performs to achieve a certain goal. They are used in many fields, such as industry, robotics, or defense, to perform complex and repetitive tasks more efficiently, quickly, and precisely, while reducing costs and risks of errors[1].

I.4 Linear and Non-Linear System

I.4.1 Linear System :

When discussing a linear system, the focus is on its response to inputs. Imagine a hypothetical machine, named $SystemS$ which performs tasks based on given instructions. This machine adheres to two key principles :

1. Homogeneity : If the instruction's magnitude increases or decreases, the system's output changes proportionally. In other words, increasing the workload (input) yields a proportional increase in output.
2. Additivity : When given separate tasks, the machine produces individual results, combining these tasks results in a collective task, with the machine's output mirroring the sum of the individual outputs. Linear control systems abide by these principles, exhibiting predictable responses to changes in task magnitude and combining tasks.

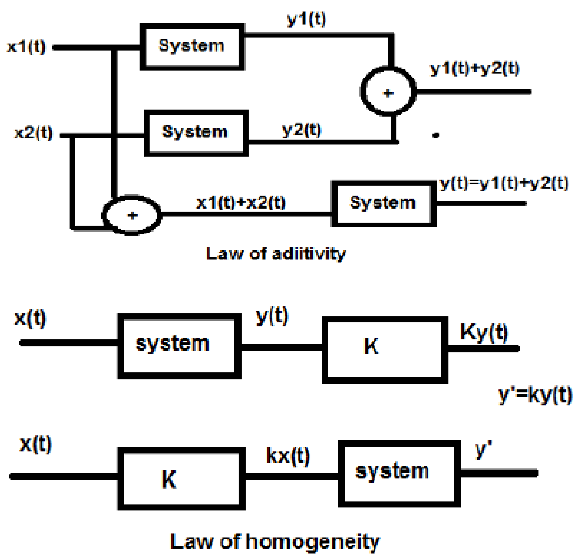

FIGURE I.1 — Linear system principals.

I.4.2 Non-Linear System :

A non-linear system deviates from the aforementioned simple rules. Unlike linear systems, where changes in one aspect produce proportional changes elsewhere, real-world systems tend to be more complex [2].

Consider a DC machine's magnetization curve, illustrating how the magnetic field varies with electrical input. Initially, increasing electricity leads to a proportional increase in the magnetic field, representing the linear phase. However, beyond a certain point, the system behaves differently. Despite continued increases in electricity, the magnetic field doesn't follow a linear trend but saturates, akin to a sponge reaching maximum absorption capacity.

Non-linear systems exhibit varied behavior, departing from straightforward, predictable patterns over their operational range. While they may initially conform to rules, they eventually diverge, displaying unexpected behaviors, as seen in the magnetization curve of a DC machine.

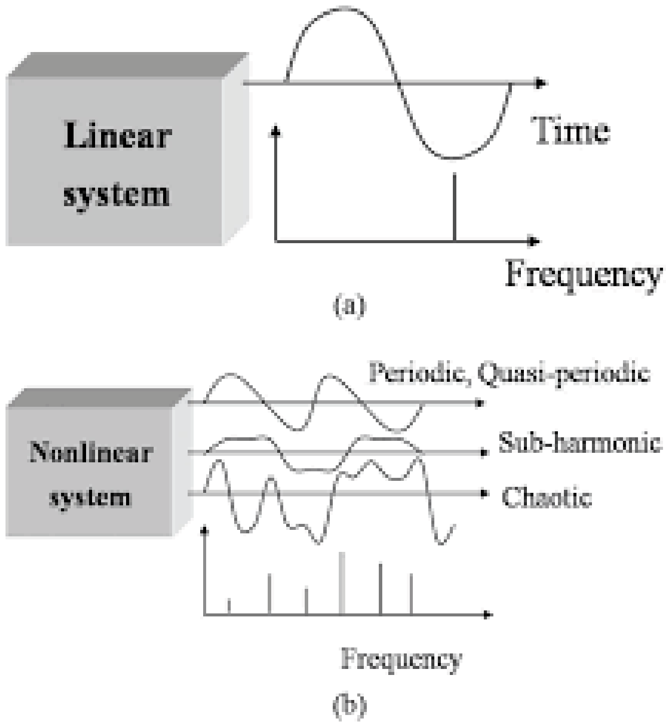

Figure I.2 — Output possibilities of (a) a linear system and (b) a nonlinear system.

I.5 Static and dynamic systems

Static System : A static system is one whose output depends only on the present input, and not on the previous inputs or outputs. The system has no memory and the

relationship between input and output is an algebraic equation.
Dynamic System : A dynamic system is a system whose output depends not only on the present input but also on the past inputs and/or outputs. This means that the resulting output can vary depending on the preceding input values and the amount of time that has elapsed since they were used. Such a system has memory and the relationship between input and output is governed by differential equations. The key difference is that in a static system, the output is determined solely by the present input, with no dependence on past inputs or outputs. In a dynamic system, the current output depends on both the current input as well as past inputs and/or past outputs of the system. Dynamic systems have a "memory" effect from prior states, this dynamic nature introduces effects like lag, resonance or integration over time. In this study, we employ the state space framework to model dynamic systems, this involves utilizing a state transition model, which delineates how states evolve over time, and a measurement model, which establishes the relationship between measurements and states. The system is mathematically described as follows :

$$\dot{x}(t) = f(x(t), u(t), \theta(t))$$
$$y(t) = h(x(t), u(t), \zeta(t))$$

Here, f represents the state transition function governing the evolution of states over time, while h denotes the measurement function connecting measurements to states, x is the vector of state variables, u is the vector of input or control variables, and θ and ζ represent unknown or uncertain parameters, y represents the measurement vector.

I.5.1 Dynamic System Analysis

This phrase refers to two very important concepts in the analysis and control of dynamic systems, namely controllability and observability.
Controllability refers to the ability to transfer a system from any initial state to any desired final state within a finite time interval, using an admissible control input. In other words, a system is controllable if it is possible to find a control input that can drive the system to a specific state, regardless of its initial conditions.
Observability on the other hand, refers to the ability to determine the system's current state from its output measurements over a finite time interval. A system is observable if it is possible to reconstruct the system's internal state from the knowledge of its inputs and outputs over a finite time period [3].

These concepts are particularly relevant in the context of state-space representations and modern control theory, where the controllability and observability of a system are

analyzed using certain conditions related to its state-space representation. The rules for controllability and observability are as follows :
Controllability : A linear time-invariant (LTI) system represented in state-space form :

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{I.1}$$

$$y(t) = Cx(t) + Du(t) \tag{I.2}$$

is controllable if and only if the controllability matrix :

$$C = \begin{bmatrix} B \; AB \; A^2B \; \cdots \; A^{(n-1)}B \end{bmatrix} \tag{I.3}$$

has full rank, where n is the order of the system (i.e., the dimension of the state vector x). In other words, the system is controllable if the controllability matrix C has rank n, which means that its columns are linearly independent.
Observability : For the same LTI system in state-space form, the system is observable if and only if the observability matrix :

$$O = \begin{bmatrix} C^T \\ (CA)^T \\ (CA^2)^T \\ \vdots \\ (CA^{(n-1)})^T \end{bmatrix} \tag{I.4}$$

has full rank n, where n is the order of the system.the system is observable if the observability matrix O has rank n, which means that its rows are linearly independent.

Both controllability and observability are essential properties for the analysis and control of dynamic systems. If a system is not controllable, it means that there are certain states that cannot be reached, which may limit the system's performance or prevent it from achieving specific objectives. Similarly, if a system is not observable, it becomes challenging to determine its internal state accurately, which can hinder the design of effective control strategies.

I.6 Modeling and identification of dynamic systems

I.6.1 Notion of a model

A model is a mathematical or computational representation of a real-world system, process, or phenomenon. It aims to capture the essential characteristics and behavior of the system using equations, algorithms, or other mathematical constructs. A model is a simplified and abstract representation of reality, designed to facilitate understanding, analysis, and prediction.

I.6.2 Modeling

Modeling is the process of creating a model that accurately represents a real-world system or process. It involves identifying the relevant variables, formulating mathematical equations or computational algorithms, and validating the model's predictions against observed data or known behavior. Modeling is an iterative process that may involve refining or updating the model based on new information or improved understanding.
Types of Models :

1. Theoretical or White-Box Models : These models are derived from fundamental principles, laws of physics, or well-established theories. They are based on prior knowledge and understanding of the system's behavior.
2. Empirical or Black-Box Models : These models are constructed solely from observed input-output data, without relying on prior knowledge of the underlying physical principles or mechanisms.
3. Grey-Box Models : These models combine elements of both theoretical and empirical models, incorporating prior knowledge and observed data to obtain a more accurate representation of the system.

I.6.3 Identification

Identification is the process of determining or estimating the parameters of a model from observed data. It is a crucial step in modeling, as it allows the model to be tailored to accurately represent the specific system or process under study. Identification involves various techniques, such as least-squares estimation, maximum likelihood estimation, or other optimization methods [4].

Important Tasks in Identification :

1. Experiment Design : Designing informative experiments or data collection procedures to ensure sufficient information for accurate parameter estimation.
2. Model Structure Selection : Choosing an appropriate model structure (e.g., linear, nonlinear, static, dynamic) that can accurately represent the system's behavior.
3. Parameter Estimation : Estimating the unknown parameters of the chosen model structure using optimization techniques and observed data.

4. Model Validation : Evaluating the identified model's accuracy and predictive capabilities using different validation techniques, such as residual analysis, cross-validation, or statistical hypothesis testing.
5. Model Refinement : Attentively refining the model structure or parameter estimates based on validation results or new data to improve the model's accuracy and predictive power.

I.7 Stability of linear and non linear systems

Stability is a fundamental concept in the analysis and control of both linear and nonlinear systems. However, the methods and techniques used to assess stability differ between linear and nonlinear systems. Here's an overview of stability analysis :
Stability of Linear Systems : For linear time-invariant (LTI) systems, stability analysis is well-established and can be performed using various methods, including :

1. Eigenvalue or Pole Analysis : The system is stable if and only if all eigenvalues (poles) of the system matrix A have negative real parts, i.e., they lie in the left-half of the complex plane.
2. Routh-Hurwitz Criterion : This algebraic method involves analyzing the coefficients of the characteristic equation associated with the system matrix A to determine stability.
3. Nyquist Criterion : This frequency-domain method involves analyzing the Nyquist plot of the system's open-loop transfer function to determine the number of unstable poles.
4. Lyapunov's Indirect Method : This technique involves constructing a positive-definite Lyapunov function and analyzing its time derivative to establish stability.

Stability of Nonlinear Systems : Stability analysis for nonlinear systems is more complex and often relies on advanced mathematical tools and techniques, including :

1. Lyapunov's Direct Method : This is a powerful and widely used technique that involves constructing a suitable Lyapunov function and analyzing its properties to establish stability. It can be used to determine both local and global stability.
2. Describing Function Analysis : This method approximates the nonlinear system with a quasi-linear model and analyzes the stability of the resulting linear system.
3. Phase Plane Analysis : This graphical technique involves analyzing the phase portraits and trajectories of the nonlinear system in the state space to determine stability.

4. Linearization and Perturbation Methods : These techniques involve linearizing the nonlinear system around an equilibrium point or a nominal trajectory and analyzing the stability of the linearized system.

5. Energy Methods : These methods involve analyzing the energy or dissipation properties of the nonlinear system to establish stability.

It's important to note that for nonlinear systems, stability can be classified as local or global, and the specific techniques employed depend on the system's complexity and the desired level of analysis [5].

I.8 Automatic Control Theory

The field of automatic control theory focuses on the purposeful regulation of a system's dynamics. Across numerous industrial sectors and equipment, it is paramount to keep critical physical variables at prescribed levels, withstanding any fluctuations or disturbances, whether originating from within the system or from external sources, that might otherwise cause deviations from the desired values.

Open-Loop Control Systems : Open-loop control systems are those in which the output has no influence on the control action. In other words, in an open-loop control system, the output is neither measured nor fed back for comparison with the input. A practical example of an open-loop control system is a washing machine, where the soaking, washing, and rinsing cycles operate on a predetermined time basis, without measuring the actual cleanliness of the clothes (the output signal).
In any open-loop control system, the output is not compared to the reference input. Consequently, for each reference input, there is a fixed operating condition, and the system's accuracy depends on its calibration. If disturbances are present, an open-loop control system will not be able to perform the desired task accurately. Open-loop control can only be practical when the relationship between the input and output is known precisely, and there are no internal or external disturbances affecting the system. It is important to note that any control system that operates solely based on time, without feedback, is considered an open-loop system.

The major disadvantages of open-loop control systems are as follows :

1. Disturbances and changes in calibration cause errors, and the output may be different from what is desired.

2. To maintain the required quality in the output, recalibration is necessary from time to time.

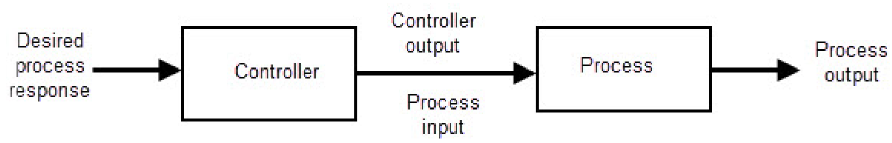

Figure I.3 — Linear system principals.

Closed-Loop Control Systems : Systems that employ feedback control are commonly referred to as closed-loop control systems. In practical applications, the terms "feedback control" and "closed-loop control" are used interchangeably. In a closed-loop control system, the error signal, which is the difference between the input signal and the feedback signal (either the output signal itself or a function derived from the output signal and its derivatives and/or integrals), is fed back to the controller. The controller then acts to reduce this error and bring the system's output to the desired value. The use of feedback control action to minimize system error is an inherent characteristic of closed-loop control systems [6].

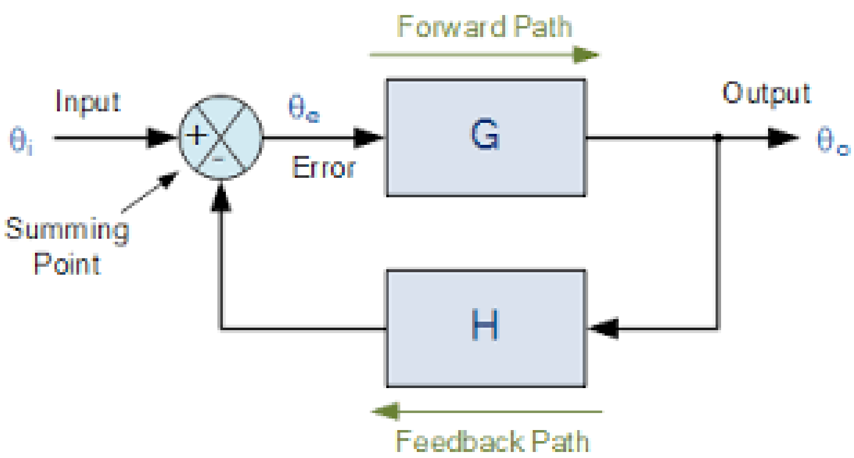

Figure I.4 — Closed-Loop Control Systems

I.8.1 Regulation

Regulation and servomechanisms (or control and servomechanisms) is a branch of automation engineering that deals with the design and analysis of control systems.Regulation refers to the process of maintaining a specific output or state within a system, while servomechanisms (or servos) are devices or mechanisms that use feedback control to maintain a desired output or position.

I.8.2 Instrumentation

Instrumentation is the study and application of measurement and control techniques, encompassing the design, development, and implementation of instruments and systems for acquiring, processing, and analyzing data. It involves the use of sensors, transducers, signal conditioning circuits, data acquisition systems, and control systems to measure and control physical quantities such as temperature, pressure, flow, level, humidity, and many others.

I.8.3 Piping and instrumentation diagram (P & ID)

A piping and instrumentation diagram (P & ID) is a detailed diagram used in process industries to represent the interconnection of piping, instrumentation, and major equipment in a plant or processing facility. It provides a comprehensive overview of the entire process, including the flow paths of materials, the location and purpose of instrumentation, and the interconnections between various components. Here's a more detailed explanation of a piping and instrumentation diagram :

1. Piping : The P&ID shows the arrangement and interconnections of pipes, including their sizes, materials, and flow directions. It depicts the flow paths of liquids, gases, or other process materials through the plant.
2. Instrumentation : The diagram clearly identifies and locates all instrumentation devices, such as sensors, transmitters, controllers, and actuators, within the process. These instruments are represented by standard symbols and are labeled with their tag numbers and descriptions.
3. Equipment : Major equipment, such as pumps, compressors, vessels, heat exchangers, and reactors, are represented by their respective symbols on the PID. Their connections to the piping system and associated instrumentation are shown.
4. Process data : The PID typically includes important process data, such as flow rates, temperatures, pressures, and compositions, at various points in the system.
5. Control loops : The diagram illustrates the relationships between instruments and the control loops they form, allowing for the monitoring and control of process variables.
6. Safety and regulatory devices : Safety devices, such as relief valves, rupture discs, and emergency shutdown systems, are also depicted on the PID.

PIDs are essential for the design, construction, operation, and maintenance of process plants. They serve as a comprehensive reference for plant personnel, enabling them to understand the process flow, identify potential issues, and troubleshoot problems efficiently. Additionally, PIDs are used for training purposes, risk assessments, and regulatory compliance.

I.8.4 Measurement fundamentals

I.8.4.1 Concept of Measurement :

Measuring a quantity involves comparing it with another quantity of the same kind, taken as the unit. It is the expression of any quantity, most often by a number followed by a symbol (the number expresses the value of the measured quantity, and the symbol expresses its nature, which is defined by a unit).
The measurements to be performed in an industrial environment or in university research laboratories are extremely varied. Several categories can be distinguished : simple measurements, complex measurements, and multiple measurements.

I.8.4.2 Measurement Chain :

It is the set of elements necessary to know the value or evolution of parameters of a physical system [15]. To capture a physical quantity and make it usable for a user, we use a measurement chain that includes the following elements :

1. A sensor
2. A signal conditioner that processes the signal delivered by the sensor to extract a usable signal

I.8.4.3 Measurement Error

The measurement error is the difference between the measured value of a quantity and a reference value. For a measurement to be complete, it must include the measured value and the limits of the possible error on the given value.
Classification of Errors : Depending on the causes there exist two types of errors :
Systematic errors : These are errors due to a known cause. Their causes can be : the measurement method, the operator, or the measuring instrument.
Random errors : These are errors that do not obey any known law when taken on a single result. They obey the laws of statistics when the number of results becomes very large. They can originate from the operator, the instrument, or the setup.

I.9 Sensors and actuators used in hydraulic systems

Sensors and actuators are essential components in a hydraulic system, Where sensors provide feedback on the system's operating conditions by measuring and monitoring various parameters in a hydraulic system, such as pressure, flow rate, temperature, position, and fluid level.On the other hand, actuators are responsible for carrying out the desired actions based on the input from the hydraulic power source and control system. The proper selection, placement, and integration of sensors and actuators are crucial for ensuring the safe, efficient, and precise operation of hydraulic systems in various applications [7] .

I.9.1 Sensors

I.9.1.1 Pressure sensors

They measure the pressure of the hydraulic fluid in different parts of the system, such as pipes, cylinders, or accumulators. Common types include :
Piezoresistive Sensors : These sensors use a piezoresistive element that changes resistance proportionally to the applied pressure. They are highly accurate and widely used in hydraulic systems.
Diaphragm Sensors : They consist of a diaphragm that deforms under pressure, and this deformation is measured using strain gauges or capacitive elements.
Bourdon Tube Sensors : They use a curved, hollow tube that straightens or twists in response to pressure changes. The movement is typically measured by a mechanical linkage or electronic transducer.

I.9.1.2 Flow sensors

They measure the volumetric flow rate of the hydraulic fluid in the pipes. Common types are :
Turbine Flow Meters : These sensors have a rotary turbine that spins proportionally to the fluid flow rate. The rotational speed of the turbine is measured and converted to a flow rate.
Vortex Flow Meters : They rely on the principle of vortex shedding, where vortexes are formed downstream of an obstruction in the flow. The frequency of vortex shedding is proportional to the flow rate.
Electromagnetic Flow Meters : They operate based on Faraday's law of electromagnetic induction. The flow of conductive fluid through a magnetic field induces a voltage proportional to the flow rate .

I.9.1.3 Level sensors

Level sensors are a type of sensor used in hydraulic systems to monitor and measure the level of hydraulic fluid in tanks or reservoirs.Here are some common types of level sensors used in hydraulic systems :
Float Sensors : They use a float that rises or falls with the liquid level, and this vertical movement is detected by various mechanisms, such as mechanical linkages or magnetic sensors.

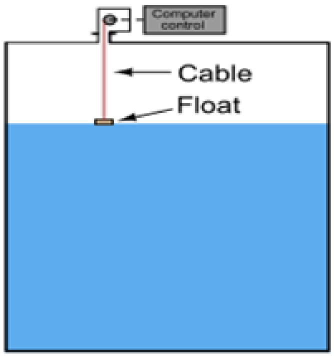

FIGURE I.5 — Float Sensor

Ultrasonic Sensors : They emit high-frequency sound waves and measure the time it takes for the reflected waves to return, which is proportional to the distance to the liquid surface.
Radar Sensors : They operate similarly to ultrasonic sensors but use electromagnetic waves instead of sound waves to measure the liquid level.

I.9.1.4 Temperature Sensors

They measure the temperature of the hydraulic fluid, which is important for controlling viscosity and preventing fluid degradation . Two types are the most commonly used are :
Thermocouples : They consist of two dissimilar metal wires joined at one end. The temperature difference between the junction and the free ends generates a small voltage proportional to the temperature.
Resistance Temperature Detectors (RTDs) : They rely on the change in electrical resistance of a pure metal (typically platinum) as its temperature changes.

I.9.1.5 Position Sensors

Position sensors are critical components in hydraulic systems as they provide feedback on the position or displacement of hydraulic actuators, such as cylinders and motors. This

information is essential for precise control and monitoring of the system's movements.Here are some common types of position sensors used in hydraulic systems :
Linear Encoders : They use a linear scale and a readhead to measure the linear position of a hydraulic cylinder or other linear motion component.
Rotary Encoders : They measure the rotational position or speed of a shaft or motor by detecting the rotation of a coded disk or scale.
Wire Displacement Sensors : They use a wire connected to the moving component, and the displacement of the wire is measured using a potentiometer or other transducer.

I.9.2 Actuators

Actuators are devices that convert energy from one form (typically electrical, hydraulic, or pneumatic) into motion or mechanical force. They are essential components in control systems and are responsible for implementing the desired action or movement based on the control signals received.
In the context of hydraulic systems, actuators are the components that convert the hydraulic energy (pressure and flow) into mechanical motion or force. The main types of hydraulic actuators are :

I.9.2.1 Hydraulic cylinders

These are linear actuators that convert hydraulic power into linear motion. They can be single-acting or double-acting and are used for lifting, pushing, or pulling loads.

— Single-acting Cylinders : They have a single port for fluid entry and rely on an external force (e.g., gravity or a spring) for the return stroke.

— Double-acting Cylinders : They have two ports, one for extending the piston and another for retracting it, providing controlled motion in both directions.

I.9.2.2 Hydraulic Motors

These are rotary actuators that convert hydraulic power into rotational motion. They are used to drive pumps, compressors, or other rotating machinery.The most common ones are :
Gear Motors : They use meshing gears to convert the hydraulic pressure and flow into rotational motion.
Vane Motors : They have a slotted rotor with vanes that slide in and out, converting the hydraulic pressure into rotational motion.

Piston Motors : They use reciprocating pistons arranged radially to convert the hydraulic pressure into rotational motion.

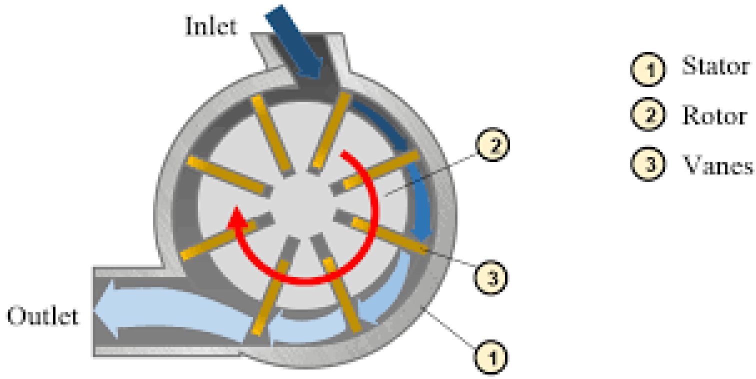

Figure I.6 — Hydraulic vane motor.

I.9.2.3 Hydraulic valves

Hydraulic valves have the main role of the control of flow, direction, and pressure of the hydraulic fluid in the system. Common types are control valves, directional valves, sequence valves, and safety valves .
Control Valves : They regulate the flow rate, pressure, or direction of the hydraulic fluid, such as flow control valves, pressure relief valves, and check valves.
Directional Valves : They control the direction of fluid flow, such as spool valves and poppet valves, allowing for the extension or retraction of hydraulic cylinders.
Sequence Valves : They ensure that specific operations occur in a predetermined order by controlling the sequence of fluid flow.
Safety Valves : They protect the system from excessive pressure buildup, such as relief valves and burst discs.

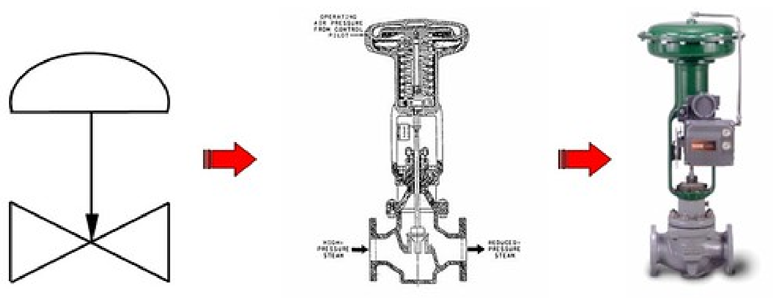

Figure I.7 — Control valve and symbolic representation.

I.9.2.4 Hydraulic pumps

The pump are instruments that provide the necessary flow and pressure for the operation of the hydraulic system. Common types are gear pumps, vane pumps, and piston pumps.
Gear Pumps : They use meshing gears to displace and transfer hydraulic fluid, creating a continuous flow.
Vane Pumps : They have a slotted rotor with vanes that slide in and out, drawing in and expelling fluid as the rotor rotates.
Piston Pumps : They use reciprocating pistons to draw in and expel hydraulic fluid, creating an intermittent flow. These sensors and actuators work together in a hydraulic system, with the sensors providing feedback to the control system, which then adjusts the actuators accordingly to maintain the desired operating conditions.

I.10 Conclusion

This chapter have provided a comprehensive overview of control systems by explaining the concepts of dynamic systems, modeling, and identification, while introducing the notions of open-loop and closed-loop control, as well as regulation and servomechanism behaviors. Additionally, it expressed an overview of sensors and actuators used in hydraulic systems.Where it has explored the different types of sensors employed to measure critical parameters such as pressure, flow, level, temperature, and position. As well as the various hydraulic actuators responsible for converting hydraulic energy into mechanical motion and force.
Overall, this chapter provided a comprehensive understanding of the critical components like sensors and actuators as well as techniques and concepts that enable the monitoring, control, and execution of desired actions in hydraulic systems across various industrial and research applications.

Chapter II

MODELING AND CONSTRUCTING TS MODELS OF A NON LINEAR SYSTEM

II.1 Introduction

The majority of physical dynamical systems encountered in reality exhibit nonlinear behavior and cannot be adequately described by linear differential equations. Linear control methods, predicated upon the assumption of a small operating range obtained from linearizing nonlinear systems. But when confronted with large operating ranges, a linear controller is prone to be unstable, because the nonlinearities in the plant cannot be properly dealt with. Moreover, linear control relies on the assumption that the system model is indeed linearizable and the linear model is accurate enough for building, so as the assumption of accurate linearization and precise knowledge of system parameters, which is often impractical for highly nonlinear systems such as mechanical and electrical systems. In cases where mathematical modeling is challenging, particularly for nonlinear systems control design is constrained by limited access to input-output data for parameter estimation. Uncertainties in model parameters further compound control challenges, potentially leading to performance degradation or instability.
To address these complexities, a nonlinear modeling approach comprising simpler submodels which are simple, understandable, and responsible for respective sub-domains, has been proposed, with fuzzy modeling utilizing fuzzy sets theory offering a novel technique for multi-model construction based on input-output data or original mathematical models. The Takagi-Sugeno fuzzy model, characterized by fuzzy IF-THEN rules representing local input-output relations, employs linear system models to capture local dynamics, culminating in an overall fuzzy model achieved through blending these linear models.

II.2 Multi-model approach

The multi-model approach is a technique for modeling (or controlling) industrial processes that exhibit inherent non linearity, have a wide operating range, or are subject to load disturbances. This approach is based on the "divide-and-conquer" strategy, where the overall problem is divided into smaller, more manageable sub problems (sub models). Then by combining the solutions of these sub-problems the overall complex system can be accurately represented.
There are two main families of multi-model structures :

1. Coupled Multi-Model Structure (Takagi-Sugeno) : In this structure, all the submodels share a common global state space. The global state vector x(t) is a weighted sum of the states of the local models. The representation of the coupled multi-model is obtained by interpolating "r" local linear models as shown in equation :

$$\dot{x}(t) = \sum_{i=1}^{r} w_i(z_C(t))[A_i x(t) + B_i u(t) + D_i u(t)]$$
$$\mathrm{y(t) = Cx(t) + Du(t)}$$

Here, $w_i(z_C(t))$ are the weighting functions that determine the contribution of each local model to the global model. This structure is the most commonly used and is also known as the Takagi-Sugeno multi-model, local parameter networks, or coupled-state multi-model.

2. Decoupled Multi-Model Structure : In this structure, each sub-model has its own independent state space. There is no common global state vector shared among the sub-models. The global model is obtained by interpolating decoupled sub-models, as shown in equation :

$$\dot{x}_i(\mathrm{t}) = \sum_{\mathrm{i=1}}^{\mathrm{i}} w_{\mathrm{i}}(\mathrm{z(t)})(\mathrm{A_i x_i(t) + B_i u(t)})$$
$$\mathrm{y_i(t) \;=\; C_i\; x_i(t)\; + D_i\; u(t)}$$

The decoupled structure can be viewed as a parallel connection of weighted affine models, This structure was initially proposed by each sub-model evolves independently in its own state space, and their outputs $y_i(t)$ are weighted to obtain the overall output. These $y_i(t)$ are artificial modeling signals used solely to describe the nonlinear behavior of the real system.

In summary, the coupled multi-model structure uses a shared global state space, while the decoupled structure allows each sub-model to have its own independent state space. The choice between these two structures depends on the nature of the nonlinear system being modeled and the desired level of complexity in the multi-model representation, The major benefits of the multi-model strategy is that it allows the use of well-established linear system theories and techniques for the design and analysis of the local models. By breaking the nonlinear or time-varying problem into smaller linear sub models, we can leverage the rich body of knowledge and tools available for linear systems.

II.3 Takagi Sugeno Fuzzy models

II.3.1 Fuzzy systems overview

A fuzzy system involves the process of establishing a mapping from given inputs to outputs using fuzzy logic. This mapping serves as a foundation for decision-making and pattern recognition. Fuzzy inference has been successfully applied across various domains including automatic control, data classification, decision analysis, expert systems, time series prediction, robotics, and pattern recognition. Due to its interdisciplinary nature, the fuzzy inference system is referred to by several names such as fuzzy-rule-based system, fuzzy expert system, fuzzy model, fuzzy associative memory, fuzzy logic controller, and simply, albeit ambiguously, fuzzy system. Fuzzy Inference System (FIS) serves to interpret input vector values and, guided by a series of fuzzy rules, allocate corresponding values to the output vector. It functions as a means of mapping input to output through the utilization of fuzzy logic. Through this mapping process, the system facilitates decision-making and pattern recognition [8].

The Mamdani fuzzy rules (systems) : pioneered by Ebhasim Mamdani, initially aimed at controlling a combination of a steam engine and boiler by employing a set of linguistic control rules derived from experienced human operators. In the Mamdani inference system, each rule's output is defined as a fuzzy logic set. In a Mamdani fuzzy system, the knowledge base consists of a set of fuzzy rules in the form :

IF (Antecedent 1) AND (Antecedent 2) AND ... THEN (Consequent)

Where both the antecedents (premises) and consequent (conclusions) of the rules are expressed using fuzzy sets and linguistic variables, which allow for the representation of imprecise or vague information. These fuzzy rules are designed to capture knowledge or observations about the system being modeled or controlled.
Takagi Sugeno fuzzy models : The TS fuzzy model, first introduced by Takagi and Sugeno in 1985, comprises an if-then rule base. In this model, the antecedents of the rules divide a portion of the model variables into fuzzy sets. Each rule's consequent is represented by a straightforward functional expression. The description of the ith rule is as follows :

$$\text{If } z_1 \text{ is } Z_{1i} \text{ and } \ldots \text{ and } z_p \text{ is } Z_{pi}, \text{ then } y = F_i(z).$$

where the vector z has p components, such j = 1, 2, ..., p, and stands for the vector of antecedent variables ; these variables are also called scheduling variables, as their values determine the degree to which rules are active. The sets Zj i, j = 1, 2, ..., p, i = 1, 2, ..., m,are the antecedent fuzzy sets , where m is the number of rules . The value of a scheduling variable z_j belongs to a fuzzy set Z_j^i with a truth value given by the membership function $\omega_{ij} : \mathbb{R} \rightarrow [0,1]$. The truth value for an entire rule is determined based on the individual premise variables, using a conjunction operator such as the minimum [9] .

$$\varphi_i(z) = \min_j \{\omega_{ij}(z_j)\}$$

or the algebraic product

$$\varphi_i(z) = \prod_{j=1}^{p} \omega_{ij}\left(z_j\right)$$

The obtained truth value is normalized

$$w_i(z) = \frac{\varphi_i(z)}{\sum_{j=1}^{m} \varphi_j(z)}$$

with conditioning that $\sum_{j=1}^{m} \varphi_j(z) \neq 0$, i.e., that for any combination of the scheduling variables at least one rule has a truth value greater than zero.So $w_i(z)$ is now referred to as the normalized membership function.

The output of rule i is determined by the consequent vector function $\mathbf{F}_i$,typically depends upon scheduling variables ;The model's output $\boldsymbol{y}$ is calculated as a weighted sum of the outputs from all rules. By using $w_i(\boldsymbol{z})$, he model's output can be represented as a function of z expressed as :

$$\boldsymbol{y} = \sum_{i=1}^{m} w_i(z)\mathbf{F}_i(z)$$

In general, the consequents of the rules (the functions $\mathbf{F}_i$) may also depend on exogenous variables, i.e., on variables that do not appear in the scheduling vector. In such a case, the output of the fuzzy model is given as

$$\boldsymbol{y} = \sum_{i=1}^{m} w_i(\boldsymbol{z})\mathbf{F}_i(\boldsymbol{z}, \boldsymbol{\theta})$$

where θ denotes the vector of exogenous variables and p_θ denotes the number of these variables.

II.4 Dynamic TS Fuzzy Models

Since our work focuses on dynamic non linear systems,we have to consider a TS models that enable us to represent such systems, So lets consider a dynamic system as follow :

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta}) \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}, \zeta) \end{aligned} \tag{II.1}$$

where $\boldsymbol{f}$ and $\boldsymbol{h}$ are smooth nonlinear functions, with $\boldsymbol{f}$ representing the state model and with h representing the measurement model, $\boldsymbol{x} \in \mathbb{R}^{n_+}$is the state vector, $\boldsymbol{u} \in \mathbb{R}^{n_-}$is the input vector, $y \in \mathbb{R}^{n_v}$ is the measurement vector, and θ and ζ represent vectors of constant parameters or other exogenous variables that act on the system [10].

A TS fuzzy system, which models or approximates such system is described as a collection

of m fuzzy rules structured as follows :
Model rule i : If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{f}}_i(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$$
$$\boldsymbol{y} = \hat{\boldsymbol{h}}_i(\boldsymbol{x}, \zeta)$$

where $z_j, j = 1, 2, \ldots, p$, represent the scheduling variables, and $\hat{\boldsymbol{f}}_i$ and $\hat{\boldsymbol{h}}_i$ are the consequent functions of the ith rule. The scheduling variables are usually chosen as a subset of the state, input, output, or other exogenous variables in the system, or they are functions of the states, inputs, outputs, or exogenous variables. The membership functions $\omega_{ij}(z_j)$ are chosen such that their truth values are in $[0, 1]$, and for any allowed value of z at least one of the rules is active. Then, the truth values of the rules are computed and normalized.then finally combined into :

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{m} \varphi_i(\boldsymbol{z})\hat{\boldsymbol{f}}_i(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})}{\sum_{i=1}^{m} \varphi_i(\boldsymbol{z})} = \sum_{i=1}^{m} w_i(\boldsymbol{z})\hat{\boldsymbol{f}}_i(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$$
$$\boldsymbol{y} = \frac{\sum_{i=1}^{m} \psi_i(\boldsymbol{z})\hat{\boldsymbol{h}}_i(\boldsymbol{x}, \zeta)}{\sum_{i=1}^{m} \psi_i(\boldsymbol{z})} = \sum_{i=1}^{m} w_i(\boldsymbol{z})\hat{\boldsymbol{h}}_i(\boldsymbol{x}, \zeta)$$

The consequent functions $\hat{f}_i$ and $\hat{\boldsymbol{h}}_i$ are usually less complex than the original nonlinear functions f and $\boldsymbol{h}$, and are in general chosen as constant, linear or affine functions. Since these consequents are typically valid only locally where the value of the corresponding normalized membership function is nonzero, So they will be referred to as "local models". In TS fuzzy systems with linear or affine local models, the rules will have the following form :
Model rule : If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\dot{\boldsymbol{x}} = A_i\boldsymbol{x} + B_i\boldsymbol{u}$$
$$\boldsymbol{y} = C_i\boldsymbol{x}$$

for linear models : If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\dot{\boldsymbol{x}} = A_i\boldsymbol{x} + B_i\boldsymbol{u} + a_i$$
$$\boldsymbol{y} = C_i\boldsymbol{x} + c_i$$

for affine models. In the expressions above, A_i, B_i, C_i are the matrices and a_i, c_i are the biases of the i^th local model. The final outputs of the TS system are computed as :

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{m} w_i(z)\left(A_i x + B_i \boldsymbol{x}\right) \\ \boldsymbol{y} &= \sum_{i=1}^{m} w_i(z) C_i \boldsymbol{x} \end{aligned} \tag{II.2}$$

And for models with linear consequents :

$$\begin{aligned}
\dot{\boldsymbol{x}} &= \sum_{i=1}^{m} w_i(\boldsymbol{z})\left(A_i\boldsymbol{x} + B_i\boldsymbol{u} + a_i\right)\\
\boldsymbol{y} &= \sum_{i=1}^{m} w_i(\boldsymbol{z})\left(C_i\boldsymbol{x} + c_i\right)
\end{aligned} \tag{II.3}$$

The utilization of normalized membership functions results in the linear dynamic TS model being effectively a convex combination of local linear models. This particular attribute simplifies the stability analysis of the fuzzy system.

Example 2.1. Consider the nonlinear dynamic system

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_1x_2 \quad y = x_1\\
\dot{x}_2 &= x_1 - 3x_2
\end{aligned} \tag{II.4}$$

with $x_1, x_2 \in [-1, 1]$. This system can be exactly represented (using the sector nonlinearity approach, that will be introduced later on) by the following TS fuzzy system with linear consequents .

Model rule 1 : If z_1 is around -1 then :

$$\dot{\boldsymbol{x}} = \begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix} \boldsymbol{x}$$

$$y = x_1$$

Model rule 2 : If z_1 is around 1 then :

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 0 \\ 1 & -3 \end{pmatrix} \boldsymbol{x}$$

$$y = x_1$$

In the model above, the scheduling variable z_1 is chosen as x_2, the fuzzy sets are Z_1^1 = 'around -1 ', Z_1^2 = 'around 1 ', and the corresponding membership functions are $\omega_{11} = (1 - z_1)/2$ and $\omega_{21} = (1 + z_1)/2$, respectively. It can be easily seen that with these membership functions, we have

$$\frac{1-x_2}{2}\begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1+x_2}{2}\begin{pmatrix} 0 & 0 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 + x_1x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

$$\frac{1-x_2}{2}x_1 + \frac{1+x_2}{2}x_1 = x_1 = y$$

i.e., the fuzzy model is an exact representation of the nonlinear system demonstrated earlier in this example (II.4), in the compact set $S = \{x_1, x_2 \in [-1, 1]\}$.

II.5 Constructing TS Models

There are two main approaches (strategies) to obtain Takagi-Sugeno (TS) fuzzy models :

1. Data-driven identification : In this approach, the TS fuzzy model is identified and constructed based on measured or simulated data from the system.
2. Analytical construction : This approach involves constructing a TS fuzzy model that precisely represents or approximates a given nonlinear dynamic system through analytical methods.

So far, the data-driven identification approach has been primarily applied to the development of discrete-time TS models. However, our focus is on continuous-time TS systems. Therefore, methods for identifying continuous-time TS models from data will not be covered here. Several analytical methods exist for constructing fuzzy representations or approximations of nonlinear systems . One notable method is the sector nonlinearity approach, which can be used to obtain a TS model that serves as an exact fuzzy representation of a given nonlinear system. By following the method described in [11], a TS fuzzy model can be constructed that approximates both the nonlinear system and its derivative. Another approach for approximating nonlinear systems is dynamic linearization, which essentially involves performing a Taylor series expansion around multiple operating points. In summary, TS fuzzy models can be developed either through data-driven identification or analytical construction from a known nonlinear system. While data-driven methods have focused on discrete-time models, analytical approaches like the sector nonlinearity technique and dynamic linearization allow for the approximation or exact representation of continuous-time nonlinear systems using TS fuzzy models.

II.5.1 The Sector Nonlinearity Approach

This method stands out as one of the most commonly employed strategies in building TS models for fuzzy control design. It allows us to obtain the precise fuzzy representation of a nonlinear system within a region of the state space . Originally this approach was designed for systems of the form :

$$\begin{aligned} \dot{x} &= f^m(x,u)x + g^m(x,u)u \\ y &= h^m(x,u)x \end{aligned} \tag{II.5}$$

but we will be considering the following form of non linear systems :

$$\begin{aligned} \dot{x} &= f^m(x,u)x + g^m(x,u)u + a(x,u) \\ y &= h^m(x,u)x + c(x,u) \end{aligned} \tag{II.6}$$

this expression is more broad, which makes it commonly used to obtain TS fuzzy models with linear consequents.In fact, most nonlinear dynamic system can be written in this form.

In the expression above (II.6), $\boldsymbol{f^m}$, $\boldsymbol{g^m}$, and $\boldsymbol{h^m}$ are smooth nonlinear matrix functions, $\boldsymbol{x} \in \mathbb{R}^{n_+}$ is the state vector,$\boldsymbol{u} \in \mathbb{R}^{n_+}$is the input vector, and $\boldsymbol{y} \in \mathbb{R}^{n_+}$ the measurement vector , The elements of the matrix functions $\boldsymbol{f^m}$, $\boldsymbol{g^m}$, and $\boldsymbol{h^m}$ are assumed to be bounded. Also, the variables are assumed to be defined on a compact set.

With the assumptions above, the terms of the matrix functions $\boldsymbol{f}^{\mathrm{m}}, \boldsymbol{g}^{\mathrm{m}}$, and $\boldsymbol{h}^{\mathrm{m}}$, and of the vector functions $\boldsymbol{a}$ and $\boldsymbol{c}$ are either constants or bounded. The scheduling variables are chosen as $z_j(\cdot) \in \left[\underline{\mathrm{nl}}_j, \overline{\mathrm{nl}}_j\right], j = 1, 2, \ldots, p$, where z_j represent the non-constant terms in $\boldsymbol{f}^{\mathrm{m}}, \boldsymbol{g}^{\mathrm{m}}, \boldsymbol{h}^{\mathrm{m}}, \boldsymbol{a}$, and $\boldsymbol{c}$, and $\underline{\mathrm{nl}}_j$ and $\overline{\mathrm{nl}}_j$ are the minimum and maximum [2], respectively, of z_j. Then, for each z_j, two weighting functions can be constructed as

$$\eta_0^j(\cdot) = \frac{\overline{\mathrm{nl}}_j - z_j(\cdot)}{\overline{\mathrm{nl}}_j - \underline{\mathrm{nl}}_j} \quad \eta_1^j(\cdot) = 1 - \eta_0^j(\cdot) \quad j = 1, 2, \ldots, p$$

Note that :

— These weighting functions are normalized in whiche $\eta_0^j(\cdot) \geq 0, \eta_1^j(\cdot) \geq 0$, and $\eta_0^j + \eta_1^j = 1$, for any value of z_j.

— z_j has the capacity to be articulated as $z_j = \underline{\mathrm{nl}}_j \eta_0^j\left(z_j\right) + \overline{\mathrm{nl}}_j \eta_1^j\left(z_j\right)$as the weighted sum of the two extreme.

— The fuzzy sets corresponding to both weighting functions are established upon $\left[\underline{\mathrm{n}}_j, \overline{\mathrm{nl}}_j\right]$, i.e., the domain where z_j takes its values. These fuzzy sets are represented (denoted) in the sequel by $\bar{Z}_0^j$ and $\bar{Z}_1^j$.

The rules of the TS system are devised to encompass all terms $z_j, j = 1, 2, \ldots, p$, where j ranges from 1 to p, ensuring that the rules adhere to the form :

Model rule i

If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\begin{aligned} \dot{\boldsymbol{x}} &= A_i \boldsymbol{x} + B_i \boldsymbol{u} + a_i \\ \boldsymbol{y} &= C_i \boldsymbol{x} + c_i \end{aligned}$$

where Z_j^i, i = 1, 2, …, m, j = 1, 2, …, p, can be either Z_1^i or Z_0^i. As a result the TS system consists of $m - 2^p$ rules. The membership function of rule i is computed as the product of the weighting functions that correspond to the fuzzy sets in the rule such as :

$$\omega_i(z) = \prod_{j=1}^{p} \omega_{ij}(z_j)$$

where $w_{ij}(z_j)$ is either $\eta_1^j(\cdot)(z_j)$ or $\eta_0^j(\cdot)(z_j)$, Depending on the weighting function employed within the rule. Thanks to the design of these weighting functions, the resulting membership functions are normalized $\omega_i(z) \geq 0$ and $\sum_{i=1}^{m} w_i(z) = 1$. The matrices A, B, C, and the vectors a_i and c_i are constructed by substituting the elements corresponding to the weighting functions used in rule i like $\underline{\mathrm{nl}}_j$ (min) for η_0^j and $\overline{\mathrm{nl}}_j$ (max)for η_1^j ,respectively, into the matrix and vector functions f^m, g^m, h^m, a, and c. Finally by using the membership functions given we can now represent the nonlinear system precisely by the TS fuzzy model given by :

$$\begin{aligned} \dot{\boldsymbol{x}} &= \sum_{i=1}^{m} \omega_i(z)(A_i x + B_i u + a_i) \\ \boldsymbol{y} &= \sum_{i=1}^{m} \omega_i(z)(C_i x + c_i) \end{aligned} \tag{II.7}$$

It's important to note that the general form of nonlinear systems (II.7) aren't unique, thus resulting in non-uniqueness in the TS representation of the nonlinear system obtained through the sector non-linearity approach.

The primary benefit of the sector non-linearity approach is that it offers the main advantage of providing an exact representation of the nonlinear system through the obtained Takagi-Sugeno (TS) model.

However, this approach has two significant drawbacks. Firstly, the resulting consequent linear or affine models are not guaranteed to be stable or observable (detectable), even if the nonlinear system itself possesses these properties. Most methods for analyzing the stability of TS systems require the local models to be stable. Similarly, observer design methods necessitate the local models to be observable or detectable. Depending on the specific nonlinear system under consideration, instability or unobservability of the local models may be avoidable by choosing an alternative representation of the nonlinear system. Otherwise, methods that obtain an approximate fuzzy model, whose local models share the same properties as the nonlinear system, such as the one presented in the next section, can be employed. Secondly, the number of rules, or local models, in the obtained TS model grows exponentially with the number of nonlinearities. In practical applications, a large number of local models may render the design problems computationally intractable or incompatible with current algorithmic limitations. Consequently, unless instability or unobservability of the local models is a concern, a fuzzy representation with a minimal number of rules should be preferred.

II.5.2 Linearization approach

An alternative method for acquiring a TS fuzzy approximation of a provided nonlinear model is linearization, as outlined by Johansen et al. (2000). This approach is actually a Taylor series expansion at various representative points, which may or may not be equilibrium points.

Consider the following dynamic nonlinear system where $\boldsymbol{x} \in \mathbb{R}^{n_+}$is the state vector, $\boldsymbol{u} \in \mathbb{R}^{n_-}$is the input vector, $y \in \mathbb{R}^{n_v}$ is the measurement vector $\boldsymbol{f}$ and $\boldsymbol{h}$ are smooth nonlinear functions :

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$$

The objective is to achieve an approximation of this nonlinear system through a set of m rules formatted as Model rule i : If z_1 is Z_1^i and ... and $\mathrm{z}_p is\ \mathrm{Z}_l^p$ then

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{m} w_i(\boldsymbol{z}) \left(A_i\boldsymbol{x} + B_i\boldsymbol{u} + a_i\right)$$

$$\boldsymbol{y} = \sum_{i=1}^{m} w_i(\boldsymbol{z}) \left(C_i\boldsymbol{x} + c_i\right)$$

or, equivalently, a TS model of the form

$$\dot{x} = \sum_{i=1}^{m} w_i(z)(A_ix + B_iu + a_i)$$

$$\boldsymbol{y} = \sum_{i=1}^{m} w_i(z)(C_ix + c_i)$$

where A_i, B_i, a_i, C_i, and c_i are the matrices and biases of the local linear models, z is the scheduling vector that determines which of the rules are active at a certain moment, and $w_i(z, i = 1, 2), \ldots, m$ are the normalized membership functions First,One needs to determine which variables characterize the nonlinearities and which variables should serve as the scheduling variables. This entails making decisions regarding the selection of inputs, states, and measurements for the variable z. Second, a sufficient number of m linearization points $z_{0,i}, i = 1, 2, \ldots, m$ have to be chosen, together with a partition of the space where the variables are defined . By increasing the number of well-chosen approximation points, the approximation accuracy of the fuzzy model increases. However, by increasing the number of the the linearization points, the computational costs of the controller or observer design also increase. then the consequent matrices are obtained as

$$A_i = \left.\frac{\partial f}{\partial x}\right|_{z_{0,i},0} \qquad B_i = \left.\frac{\partial f}{\partial u}\right|_{z_{0,i},0} \qquad C_i = \left.\frac{\partial h}{\partial x}\right|_{z_{0,i},0}$$

where $z_{0,i}$ represents the evaluation of the expression on the left in the value corresponding to $z_{0,i}$ for those state and input variables that are scheduling variables and 0 for those states and inputs that are not z. Because linearization is typically not conducted at equilibria, it necessitates the addition of affine terms.

$$a_i = f(x,u)|_{z_{0,i},0} - (A_i x)|_{z_{0,i},0} - (B_i u)|_{z_{0,i},0}$$

$$c_i = h(x)|_{z_{0,i},0} - (C_i x)|_{z_{0,i},0}$$

To obtain the desired TS system the membership functions of each rule are computed and normalized [9].

The advantage of constructing TS models using linearization lies in the fact that although the fuzzy system is an approximation of the original nonlinear system, the consequents retain crucial properties of the nonlinear system at the linearization points. For example, if the nonlinear system is locally observable in a vicinity of the linearization point, then the corresponding local model is also observable or detectable. However, a drawback of this method is the absence of general guidelines for selecting the linearization points or determining how many should be chosen. Depending on the nonlinearity, a considerable number of points may be required for an accurate approximation, resulting in significant computational costs. Moreover, since linearization is typically not performed at equilibrium points, this method yields affine TS models.

Consequently, stability analysis and controller design become more challenging. Nevertheless, as previously mentioned, affine models do not pose difficulties in observer design.

II.6 Conclusion

In this chapter, we have introduced the Takagi-Sugeno (TS) fuzzy models which will be used in the subsequent parts of the work. Furthermore, two methods for constructing dynamic TS models from a nonlinear dynamic system were discussed. The first method, called the sector non-linearity approach, allows for the construction of exact fuzzy representations of the nonlinear system where we presented how this method can modulate a complex nonlinear system. The second method, linearization, yields a TS model that approximates the nonlinear system. This approximation method has the advantage of retaining the local properties of the nonlinear system in the resulting TS model.

Chapter III

MODELING AND VALIDATION OF THE HYDRAULIC SYSTEM TS MODEL

III.1 Introduction

The focus of this chapter is on modeling a hydraulic system by employing a multi-model approach based on the Takagi-Sugeno fuzzy system .Where we will begin by introducing and describing the three-tank system, which will be the focus of our modeling efforts. Then, we aim to create a mathematical model that comprises multiple subsystems whose dynamics mimic the overall system's behavior. After creating the model using the Takagi-Sugeno (TS) approach, we will validate this approach by comparing the dynamic model of the system(derived from the underlying physical laws) with the model obtained through the TS approach. By comparing the results from both models, we will assess the accuracy and relevance of our work using the TS approach. This validation process will involve simulations conducted using MATLAB software. By juxtaposing the outcomes derived from the two models, the accuracy and appropriateness of the proposed multi-model approach for the studied hydraulic system can be assessed.
In simpler terms, we will first model the three-tank system using the TS approach, which combines multiple subsystems. Then, we will validate the TS model by comparing it to the dynamic model based on physical laws. This comparison, done through MATLAB simulations, will help us evaluate the accuracy of our TS model.

III.2 System description and modeling

The hydraulic nonlinear system consists of three tanks (cylinder reservoirs) Tank 1, Tank 2, and Tank 3, interconnected in series by two connecting pipes for the transfer of liquid between them as shown in (Figure 3.1).

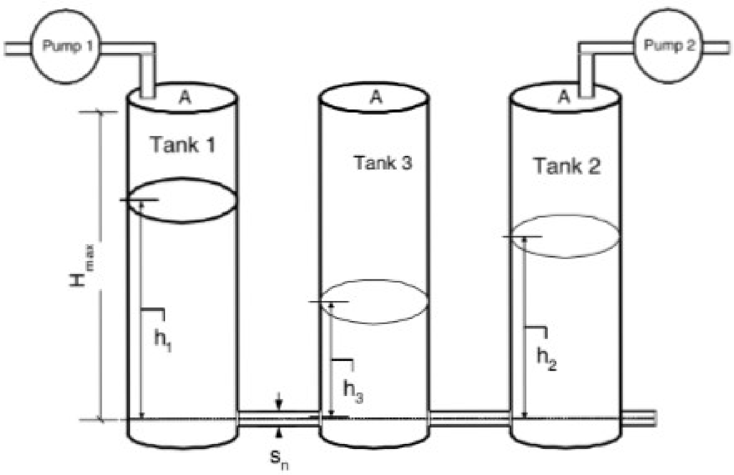

Figure III.1 — Three tank system

Pumps 1 and 2 supply the tanks T1 and T2 with a flow rate Q1 and Q2.The liquid levels (h1, h2, and h3) within the tanks can be monitored using level sensors or visual indicators, which would provide feedback on the system (both levels h1,2,3 and flow rates Q11 and Q2 are measurable). The connecting pipes and the tanks are additionally equipped with manually adjustable valves and outlets(in tank 2).
Where $h_i(t), i = 1, 2, 3$, are the water levels in each tank,Q1 and Q2 are the incoming flow rates that supply the tanks T1 and T2.
Q_{ij} is the mass flow (cm^3/s) from the ith tank to the jth tank (example : the flow rates between the tanks T1 and T3 is denoted as Q13) which are calculated using the Torricelli's law as :

$$\begin{aligned}
Q_{13} &= a_1 s_{15} \operatorname{sgn}(h_1 - h_3) \sqrt{R_g (h_1 - h_3)} \\
Q_{32} &= a_3 s_{23} \operatorname{sgn}(h_3 - h_2) \sqrt{2g)h_3 - h_2} \\
Q_{20} &= a_2 S_0 \sqrt{2gh_2}
\end{aligned} \tag{III.1}$$

System's physical parameters : Where all these parameters will be converted to (IS)

Parameters	Symbol	Value	Unit
cross section area of tanks	$\mathcal{A}$	154	cm^2
cross section area of pipes	s_n	0.5	cm^2
max. height of tanks	$H_{\max}$	62	cm
max. flow rate of pump 1	$Q_{1_{\max}}$	100	cm^3/s
max. flow rate of pump 2	$Q_{2_{\max}}$	100	cm^3/s
coeff. of flow for pipe 1	a_1	0.45	
coeff. of flow for pipe 2	a_2	0.45	
coeff. of flow for pipe 3	a_3	0.45	

III.2.1 Nonlinear Analysis(study)

The objective is to develop a nonlinear model based on an understanding of the system's behavior(knowledge model).This model will be derived from the mass balance equations for each tank (reservoir) within the system.
By applying the principles of conservation of mass and considering the inflows and outflows of fluid in each reservoir, a set of nonlinear equations will be formulated. These equations will capture the dynamic relationships between the various parameters, such as liquid

levels, flow rates, and other relevant variables. The resulting nonlinear model will provide a mathematical representation of the system's behavior, allowing us for further analysis, simulation, and to compare its behaviour with the TS model that will be constructed later on. This approach leverages the fundamental physical principles governing the system to obtain a comprehensive and accurate description of its nonlinear dynamics.

III.2.1.1 Obtaining the non linear model

The mass conservation law of a fluid is given by :

$$\rho.\frac{dV}{dt} = \rho.\Big(\sum \textbf{input flow rate} - \sum \textbf{output flow rate}\Big)$$

Where ρ is the volumetric mass density of the fluid ,and by applying this we get :
For tank T1 :

$$\rho.\frac{dV_1}{dt} = \rho.(Q_1 - Q_{13}) \tag{III.2}$$

For tank T2 :

$$\rho.\frac{dV_2}{dt} = \rho.(Q_2 + Q_{32} - Q_{20}) \tag{III.3}$$

For tank T3 :

$$\rho.\frac{dV_3}{dt} = \rho.(Q_{13} - Q_{32}) \tag{III.4}$$

Knowing that $V = A.h$, Where h is the fluid level in the reservoir and A is the cross section area of the tanks . So we get :

$$\begin{aligned} V_1 &= A.h_1 \\ V_2 &= A.h_2 \\ V_3 &= A.h_3 \end{aligned} \tag{III.5}$$

by replacing this in (III.2),(III.3) and (III.4) we get :

$$\begin{cases} A.\rho.\dot{h_1} = \rho(Q_1 - Q_{13}) \\ A.\rho.\dot{h_2} = \rho(Q_2 + Q_{32} - Q_{20}) \\ A.\rho.\dot{h_3} = \rho(Q_{13} - Q_{32}) \end{cases} \tag{III.6}$$

$A\dot{h_1}$ =Q$_1$ - Q_{13}
$A\dot{h_2}$ =Q$_2$ +Q_{32}-Q_{20} giving : $\begin{cases} \dot{h_1} = \frac{1}{A}(Q_1 - Q_{13}) \\ \dot{h_2} = \frac{1}{A}(Q_2 + Q_{32} - Q_{20}) \\ \dot{h_3} = \frac{1}{A}(Q_{13} - Q_{32}) \end{cases}$
$A\dot{h_3}$ =Q$_{13}$ - Q_{32}

(III.7)

where : $\begin{cases} Q_{13} = a_1 s_{15} \operatorname{sgn}(h_1 - h_3) \sqrt{R_g (h_1 - h_3)} \\ Q_{32} = a_3 s_{23} \operatorname{sgn}(h_3 - h_2) \sqrt{2g) h_3 - h_2} \\ Q_{20} = a_2 S_0 \sqrt{2gh_2} \end{cases}$

with the condition : $h_1 > h_3 > h_2 > 0$:

$$\begin{aligned} Q_{13} &= C_1\sqrt{h_1 - h_3} & & C_1 = \alpha_2 S_{13}(2g)^{1/2} \\ Q_{32} &= C_3\sqrt{h_3 - h_2} & \text{where :} \quad & C_2 = a_2 S_0 (2g)^{1/2} \\ Q_{20} &= C_2\sqrt{h_2} & & C_3 = a_3 S_{23}(2g)^{1/2} \end{aligned} \tag{III.8}$$

replacing this in the previous equation (III.5) :

$$\begin{aligned} \dot{h_1} &= \frac{1}{A}(Q_1 - C_1 Q_{13}) & & \dot{h_1} = \frac{1}{A}(Q_1 - C_1 sqrt h_1 - h_3) \\ \dot{h_2} &= \frac{1}{A}(Q_2 + C_2 Q_{32} - C_3 Q_{20}) & \text{where :} \quad & \dot{h_2} = \frac{1}{A}(Q_2 + C_3\sqrt{h_3 - h_2} - C_2\sqrt{h_2}) \\ \dot{h_3} &= \frac{1}{A}(C_1 Q_{13} - C_3 Q_{32}) & & \dot{h_3} = \frac{1}{A}(C_1\sqrt{h_1 - h_3} - C_3\sqrt{h_3 - h_2}) \end{aligned}$$

we finally get :

$$\begin{cases} \dot{h_1} = \dfrac{Q_1}{A} - \dfrac{C_1}{A}(\sqrt{h_1 - h_3}) \\ \dot{h_2} = \dfrac{Q_2}{A} + \dfrac{C_3}{A}(\sqrt{h_3 - h_2}) - \dfrac{C_2}{A}(\sqrt{h_2}) \\ \dot{h_3} = \dfrac{C_1}{A}(\sqrt{h_1 - h_3}) - \dfrac{C_3}{A}(\sqrt{h_3 - h_2}) \end{cases} \tag{III.9}$$

III.2.2 State space representation

Our three tanks system is considered as a multi variable system that has two input variables, u_1 and u_2, representing the flow rates Q_1 and Q_2, respectively. Additionally, the system has three output variables, y_1, y_2, and y_3, corresponding to the liquid levels h_1, h_2, and h_3 in each of the tanks,T1,T2 and T3 respectively. Such as :

The system's outputs :

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

The system's inputs :

$$u = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The state space victor :

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

Now by replacing this notions in equations (III.9) we get the following nonlinear representation :

$$\begin{cases} \dot{x_1} = \dfrac{u_1}{A} - \dfrac{C_1}{A}(\sqrt{x_1 - x_3}) \\ \dot{x_2} = \dfrac{u_2}{A} + \dfrac{C_3}{A}(\sqrt{_3 - x_2}) - \dfrac{C_2}{A}(\sqrt{x_2}) \\ \dot{x_3} = \dfrac{C_1}{A}(\sqrt{x_1 - x_3}) - \dfrac{C_3}{A}(\sqrt{x_3 - x_2}) \end{cases}$$

Giving :

$$\begin{cases} \dot{x}_1 = \dfrac{u_1}{A} - \dfrac{C_1}{A}\dfrac{\sqrt{x_1 - x_3}}{x_1}x_1 \\ \dot{x}_2 = \dfrac{u_2}{A} + \dfrac{C_3}{A}\dfrac{\sqrt{x_3 - x_2}}{x_3}x_3 - \dfrac{C_2}{A}\dfrac{\sqrt{x_2}}{x_2}x_2 \\ \dot{x}_3 = \dfrac{C_1}{A}\dfrac{\sqrt{x_1 - x_3}}{x_1}x_1 - \dfrac{C_3}{A}\dfrac{\sqrt{x_3 - x_2}}{x_3}x_3 \end{cases} \tag{III.10}$$

giving the space state representation as :

$$\dot{x} = \begin{bmatrix} -\frac{C_1}{A}\frac{\sqrt{x_1-x_3}}{x_1} & 0 & 0 \\ 0 & -\frac{C_2}{A}\frac{\sqrt{x_2}}{x_2} & \frac{C_3}{A}\frac{\sqrt{x_3-x_2}}{x_3} \\ \frac{C_1}{A}\frac{\sqrt{x_1-x_3}}{x_1} & 0 & -\frac{C_3}{A}\frac{\sqrt{x_3-x_2}}{x_3} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/A & 0 \\ 0 & 1/A \\ 0 & 0 \end{bmatrix} U \qquad \text{(III.11)}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \qquad \text{(III.12)}$$

Linear Model : After a linearization at operating point $h_1 = 45\text{cm}, h_2 = 15\text{cm}$ and $h_3 = 30$ cm, we have the following linear (nominal) model.

$$\dot{x} = Ax + Bu, \quad y = Cx$$

where :

$$A = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.0195 & 0.0084 \\ 0.0085 & 0.0084 & -0.0169 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0065 & 0 \\ 0 & 0.0065 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So in these functioning point :

$$\begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.15 \\ 0.30 \end{bmatrix}$$

The corresponding commends are :

$$\begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} = (10^{-4}) \begin{bmatrix} 0.3859 \\ 0.2082 \end{bmatrix}$$

In steady-state conditions, the system dynamics are considered to be null, which implies that the time derivative of the output variables is zero : $\dot{x}(t) = 0$. Under these circumstances, the system of equations becomes :

$$\begin{cases} 0 = \frac{u_{10}}{A} - \frac{C_1}{A}(\sqrt{x_{10} - x_{30}}) \\ 0 = \frac{u_{20}}{A} + \frac{C_2}{A}(\sqrt{x_20}) - \frac{C_3}{A}(\sqrt{x_{30} - x_{20}} \\ 0 = \frac{C_1}{A}(\sqrt{x_{10} - x_{30}}) - \frac{C_3}{A}(\sqrt{x_{30} - x_{20}}) \end{cases}$$

Where :
$$\begin{cases} u_{10} = C_1(\sqrt{x_{10} - x_{30}}) \\ u_{20} = C_2(\sqrt{x_{20}}) - (\sqrt{x_{30} - x_{20}}) \\ 0 = \frac{C_1}{A}(\sqrt{x_{10} - x_{30}}) - \frac{C_3}{A}(\sqrt{x_{30} - x_{20}}) \end{cases}$$

III.3 Constructing a TS model of the dynamic system

To apply the multi model approach, we have first transformed the system where we obtained its state-space representation while minimizing and isolating the nonlinearities that compose it as :

$$\dot{x} = \begin{bmatrix} -\frac{C_1}{A}\frac{\sqrt{x_1 - x_3}}{x_1} & 0 & 0 \\ 0 & -\frac{C_2}{A}\frac{\sqrt{x_2}}{x_2} & \frac{C_3}{A}\frac{\sqrt{x_3 - x_2}}{x_3} \\ \frac{C_1}{A}\frac{\sqrt{x_1 - x_3}}{x_1} & 0 & -\frac{C_3}{A}\frac{\sqrt{x_3 - x_2}}{x_3} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/A & 0 \\ 0 & 1/A \\ 0 & 0 \end{bmatrix} u \tag{III.13}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \tag{III.14}$$

We can now see that our nonlinear functions (The scheduling variables that are the non-constant elements in the matrix functions) take the following terms :

$$z_1(x) = \frac{\sqrt{x_1 - x_3}}{x_1}\ ; \quad z_3(x) = \frac{\sqrt{x_3 - x_2}}{x_3}\ ;\ z_2(x) = \frac{\sqrt{x_2}}{x_2}\ ;$$

Since we have $j = 3$ functions, our model will have $2^3 = 8$ subsystems,In result we will have the states vector like :

$$\dot{x} = \begin{bmatrix} -\frac{C_1}{A} z_1(x) & 0 & 0 \\ 0 & -\frac{C_2}{A} z_2(x) & \frac{C_3}{A} z_3(x) \\ \frac{C_1}{A} z_1(x) & 0 & -\frac{C_3}{A} z_3(x) \end{bmatrix} x + \begin{bmatrix} 1/A & 0 \\ 0 & 1/A \\ 0 & 0 \end{bmatrix} u \tag{III.15}$$

from here on we will be referring to the constant C_i/A as C_{ii} (ex :$C_{11} = \frac{C_1}{A}$)

$$\dot{x} = \begin{bmatrix} -C_{11} z_1(x) & 0 & 0 \\ 0 & -C_{22} z_2(x) & C_{33} z_3(x) \\ C_{11} z_1(x) & 0 & -C_{33} z_3(x) \end{bmatrix} x + \begin{bmatrix} 1/A & 0 \\ 0 & 1/A \\ 0 & 0 \end{bmatrix} u \tag{III.16}$$

We aim by using this approach to obtain a TS model of the form :

$$\begin{aligned} \dot{\boldsymbol{x}} &= \sum_{i=1}^{j} w_i(\boldsymbol{z}) \left(A_i \boldsymbol{x} + B_i \boldsymbol{u} \right) \\ \boldsymbol{y} &= C \boldsymbol{x} \end{aligned} \tag{III.17}$$

III.3.1 Calculating membership functions

By supposing that $x_1(t) \in [0.03, 0.58], x_2(t) \in [0.01, 0.56], x_3(t) \in [0.02, 0.57]$, (these born values are only initial to get us starting with the simulation, once it's started and we get z_n graphs we have to re-calibrate this values in order to get the correct membership functions). The scheduling variables are chosen as $z_j(x) \in [\beta_j, \alpha_j]$,j= $1, 2, 3$.z_j represent the non-constant terms in the previous representation and β_j and α_j are respectively the minimum and maximum of z_j. Then, for each z_j, two weighting functions can be constructed as :

$$Z_{j1}\big(z_n(x)\big) = \frac{z_j(x) - \beta_n}{\alpha_j - \beta_j} \quad \text{and} \quad Z_{j2}\big(z_n(x)\big) = \frac{\alpha_j - z_j(x)}{\alpha_j - \beta_j}$$

Moreover, z_j can be expressed as :

$z_j(x) = \alpha_j Z_{j1}\big(z_j(x)\big) + \beta_j Z_{j2}\big(z_j(x)\big)$,calculated as follow :

n	α_n	β_n	$Z_{n1}(x(t))$	$Z_{n2}(x(t))$
1	$\alpha_1 = 3.33$	$\beta_1 = 0.017$	$Z_{11}\left(z_1(x,u)\right) = \frac{z_1(x,u)-\beta_1}{\alpha_1-\beta_1}$	$Z_{12}\left(z_1(x,u)\right) = \frac{\alpha_1-z_1(x,u)}{\alpha_1-\beta_1}$
2	$\alpha_2 = 10.5$	$\beta_2 = 1$	$\mathrm{Z}_{21}(x(t)) = \frac{z_2(x,u)-\beta_2}{\alpha_2-\beta_2}$	$\mathrm{Z}_{22}(x(t)) = \frac{\alpha_2-z_2(x,u)}{\alpha_2-\beta_2}$
3	$\alpha_3 = 8$	$\beta_3 = 0.9$	$\mathrm{Z}_{31}\left(z_3(x,u)\right) = \frac{z_3(x,u)-\beta_2}{\alpha_3-\beta_2}$	$\mathrm{Z}_{32}\left(z_3(x,u)\right) = \frac{\alpha_3-z_3(x,u)}{\alpha_3-\beta_3}$

III.3.2 Fuzzy rules model :

Since we have $j = 7$ non linear function, our model will have $2^3 = 8$ fuzzy rules.

Model rule 1 :

If z_1 is Z_{11} and z_2 is Z_{21} and z_3 is Z_{31} then :

$$\dot{\boldsymbol{x}} = A_1\boldsymbol{x} + B\boldsymbol{u}$$

Where : $$\mathrm{A}_1 = \begin{bmatrix} -C_{11}\beta_1 & 0 & 0 \\ 0 & -C_{22}\beta_2 & C_{33}\beta_3 \\ C_{11}\beta_1 & 0 & -C_{33}\beta_3 \end{bmatrix}$$

Model rule 2 :

If z_1 is Z_{11} and z_2 is Z_{21} and z_3 is Z_{32} then

$$\dot{\boldsymbol{x}} = A_2\boldsymbol{x} + B\boldsymbol{u}$$

Where : $$\mathrm{A}_2 = \begin{bmatrix} -C_{11}\beta_1 & 0 & 0 \\ 0 & -C_{22}\beta_2 & C_{33}\alpha_3 \\ C_{11}\beta_1 & 0 & -C_{33}\alpha_3 \end{bmatrix}$$

Model rule 3 :

If z_1 is Z_{11} and z_2 is Z_{22} and z_3 is Z_{31} then

$$\dot{\boldsymbol{x}} = A_3\boldsymbol{x} + B\boldsymbol{u}$$

Where : $$\mathrm{A}_3 = \begin{bmatrix} -C_{11}\beta_1 & 0 & 0 \\ 0 & -C_{22}\alpha_2 & C_{33}\beta_3 \\ C_{11}\beta_1 & 0 & -C_{33}\beta_3 \end{bmatrix}$$

Model rule 4 :

If z_1 is Z_{11} and z_2 is Z_{22} and z_3 is Z_{32} then

$$\dot{\boldsymbol{x}} = A_4\boldsymbol{x} + B\boldsymbol{u}$$

$$\text{Where :} \qquad \mathrm{A}_4 = \begin{bmatrix} -C_{11}\beta_1 & 0 & 0 \\ 0 & -C_{22}\alpha_2 & C_{33}\alpha_3 \\ C_{11}\beta_1 & 0 & -C_{33}\alpha_3 \end{bmatrix}$$

Model rule 5 :

If z_1 is Z_{12} and z_2 is Z_{21} and z_3 is Z_{31} then

$$\dot{\boldsymbol{x}} = A_5\boldsymbol{x} + B\boldsymbol{u}$$

$$\text{Where :} \qquad \mathrm{A}_5 = \begin{bmatrix} -C_{11}\alpha_1 & 0 & 0 \\ 0 & -C_{22}\beta_2 & C_{33}\beta_3 \\ C_{11}\alpha_1 & 0 & -C_{33}\beta_3 \end{bmatrix}$$

Model rule 6 :

If z_1 is Z_{12} and z_2 is Z_{22} and z_3 is Z_{31} then

$$\dot{\boldsymbol{x}} = A_6\boldsymbol{x} + B\boldsymbol{u}$$

$$\text{Where :} \qquad \mathrm{A}_6 = \begin{bmatrix} -C_{11}\alpha_1 & 0 & 0 \\ 0 & -C_{22}\alpha_2 & C_{33}\beta_3 \\ C_{11}\alpha_1 & 0 & -C_{33}\beta_3 \end{bmatrix}$$

Model rule 7 :

If z_1 is Z_{12} and z_2 is Z_{21} and z_3 is Z_{32} then

$$\dot{\boldsymbol{x}} = A_7\boldsymbol{x} + B\boldsymbol{u}$$

$$\text{Where :} \qquad \mathrm{A}_7 = \begin{bmatrix} -C_{11}\alpha_1 & 0 & 0 \\ 0 & -C_{22}\beta_2 & C_{33}\alpha_3 \\ C_{11}\alpha_1 & 0 & -C_{33}\alpha_3 \end{bmatrix}$$

Model rule 8 :

If z_1 is Z_{12} and z_2 is Z_{22} and z_3 is Z_{32} then

$$\dot{\boldsymbol{x}} = A_8\boldsymbol{x} + B\boldsymbol{u}$$

$$\text{Where :} \qquad \mathrm{A}_8 = \begin{bmatrix} -C_{11}\alpha_1 & 0 & 0 \\ 0 & -C_{22}\alpha_2 & C_{33}\alpha_3 \\ C_{11}\alpha_1 & 0 & -C_{33}\alpha_3 \end{bmatrix}$$

III.3.3 defuzzification

We finally obtain this system's TS fuzzy model as :

$$\dot{x} = \sum_{i=1}^{j} w_j(z_n(x))\,(\mathrm{A_j\ x(t) + B\ u(t)})$$

where :

$$
\begin{aligned}
w_1(z(t)) &= F_{11}(z_1(x)) \times F_{21}(z_2(x)) \times F_{31}(z_3(x,)) \\
w_2(z(t)) &= F_{11}(z_1(x)) \times F_{21}(z_2(x)) \times F_{32}(z_3(x,)) \\
w_3(z(t)) &= F_{11}(z_1(x)) \times F_{22}(z_2(x)) \times F_{31}(z_3(x,)) \\
w_4(z(t)) &= F_{11}(z_1(x)) \times F_{22}(z_2(x)) \times F_{32}(z_3(x,)) \\
w_5(z(t)) &= F_{12}(z_1(x)) \times F_{21}(z_2(x)) \times F_{31}(z_3(x,)) \\
w_6(z(t)) &= F_{12}(z_1(x)) \times F_{21}(z_2(x)) \times F_{32}(z_3(x,)) \\
w_7(z(t)) &= F_{12}(z_1(x)) \times F_{22}(z_2(x)) \times F_{31}(z_3(x,)) \\
w_8(z(t)) &= F_{12}(z_1(x)) \times F_{22}(z_2(x)) \times F_{32}(z_3(x,))
\end{aligned}
\tag{III.18}
$$

III.4 Model validation

Now that we have developed our TS Fuzzy Models for the three-tank nonlinear system, it is imperative to validate them. This process involves several key steps :

1. Establishing and simulating the dynamic nonlinear system around an operational point.
2. Constructing and simulating the TS model around the same operational point.
3. Conducting a comparative analysis of the outcomes generated by both models. The model's validity is contingent upon the magnitude of the error, which should be minimal.

These calculations and simulations will be executed using Matlab Simulink to streamline the process.

III.4.1 Overall structure (block diagram)

Here we present the overall simulation blocks that contains both dynamic and TS models(figure(III.2)), by comparison (subtraction) of the outing of each model in order to Validate our TS model,in what below the simulation of each model separately .

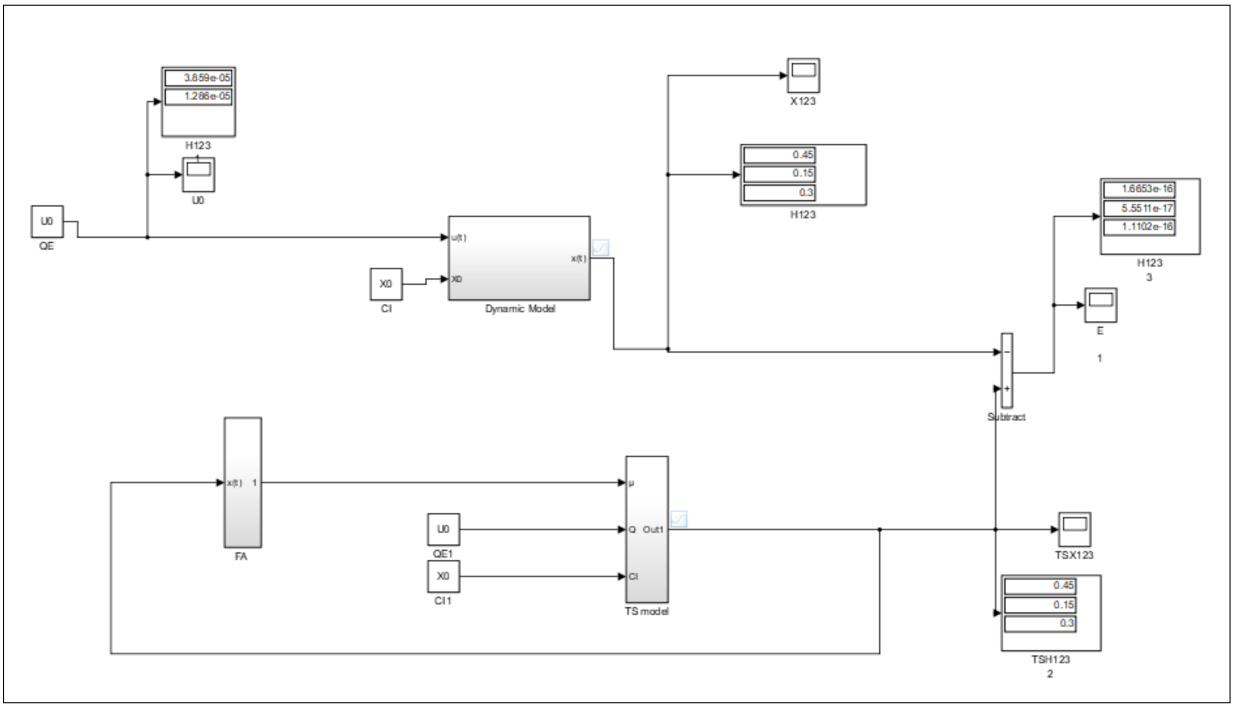

Figure III.2 — Overall simulation bloc of both TS and dynamic models

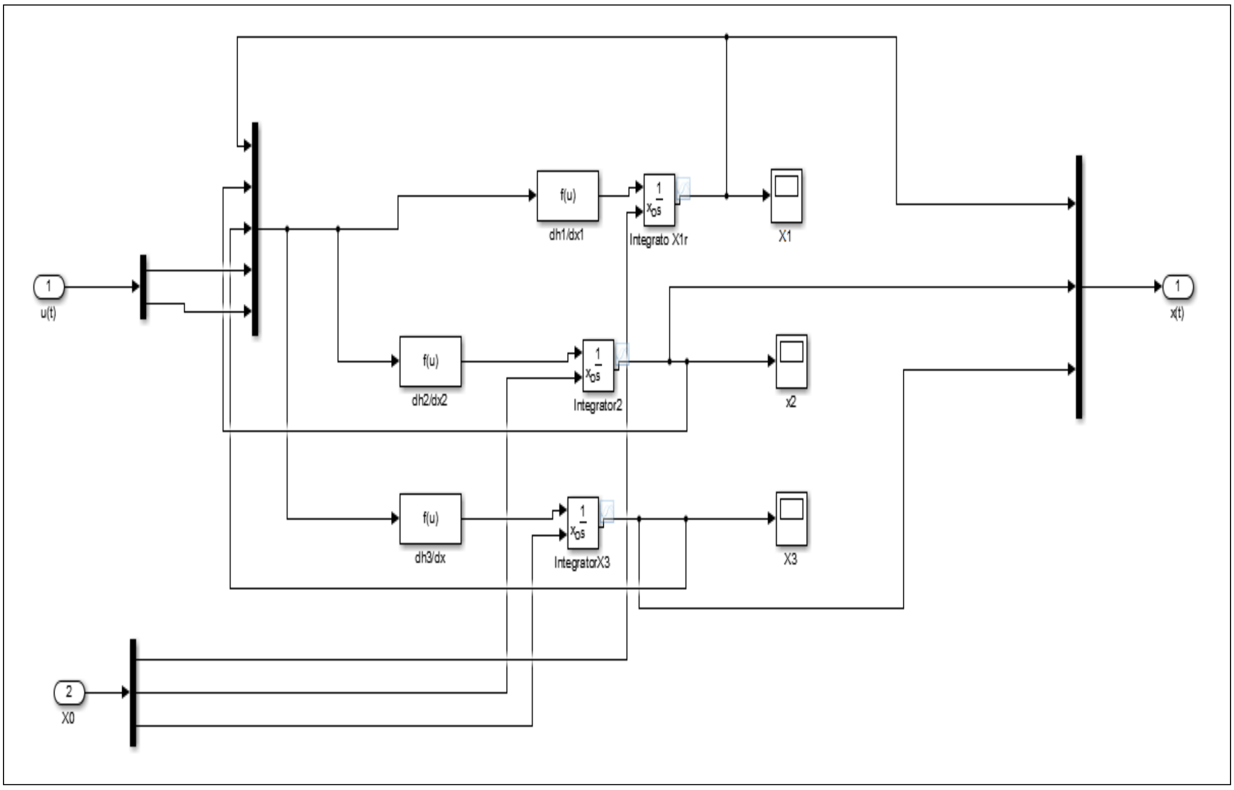

Figure III.3 — Dynamic model simulation

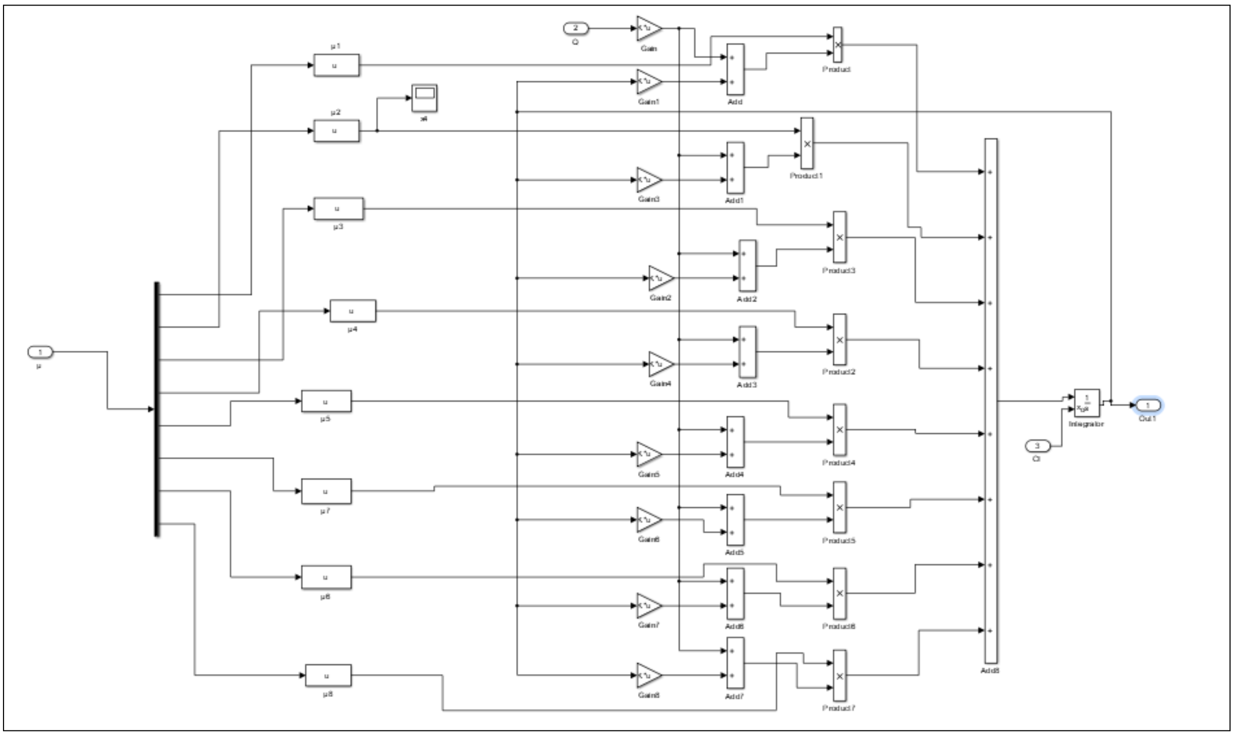

Figure III.4 — TS model simulation l

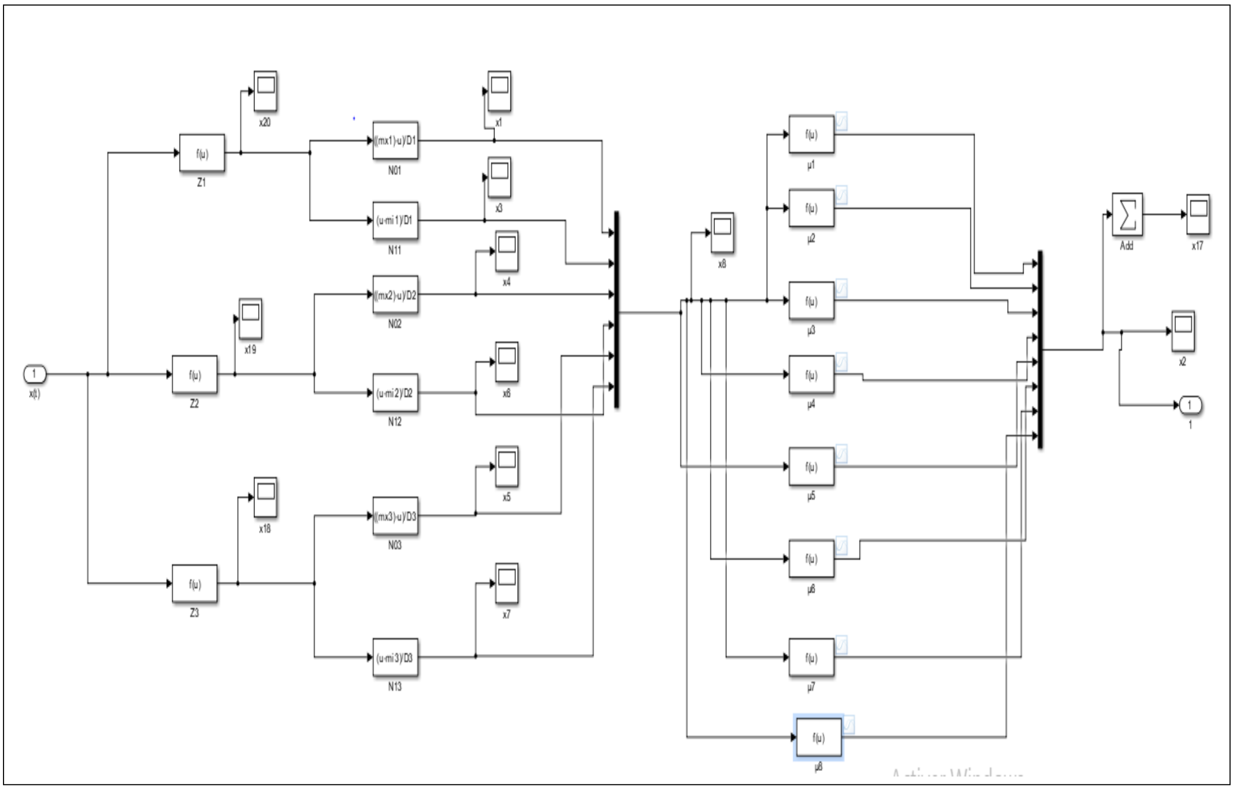

Figure III.5 — Membership function generation bloc

III.4.2 Simulation and results

III.4.2.1 First simulation :

Firstly,we will be running the simulation on the following parameters :

— Functioning point : $\begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.15 \\ 0.30 \end{bmatrix}$

— initial conditions : $X_0 = \begin{bmatrix} 0.05 \\ 0.01 \\ 0.03 \end{bmatrix}$

— Simulation step : 0.1 sec .

— Simulation time : [0 ; 5000].

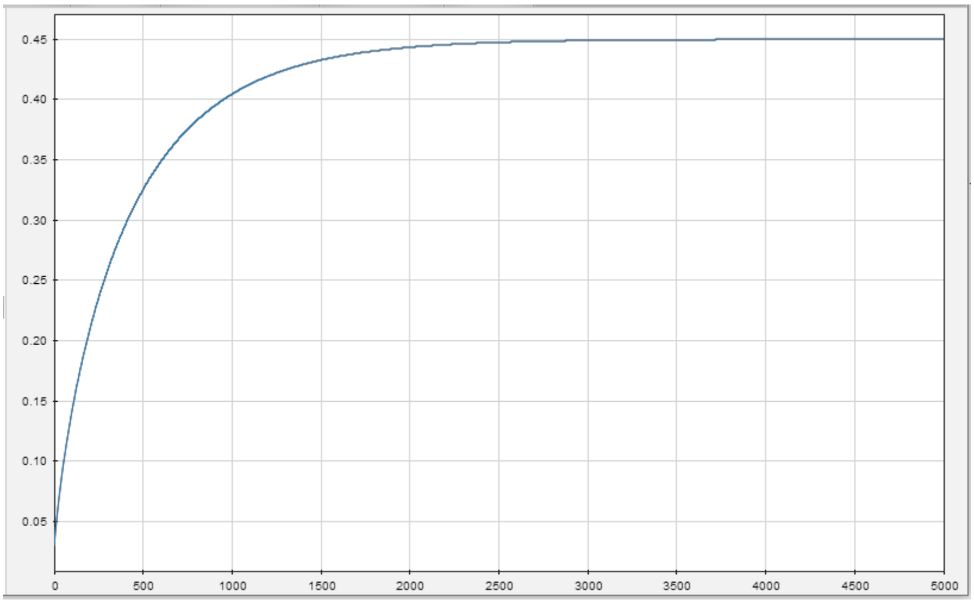

Figure III.6 — Comparison of the level H1 between the dynamic nonlinear model and the TS model

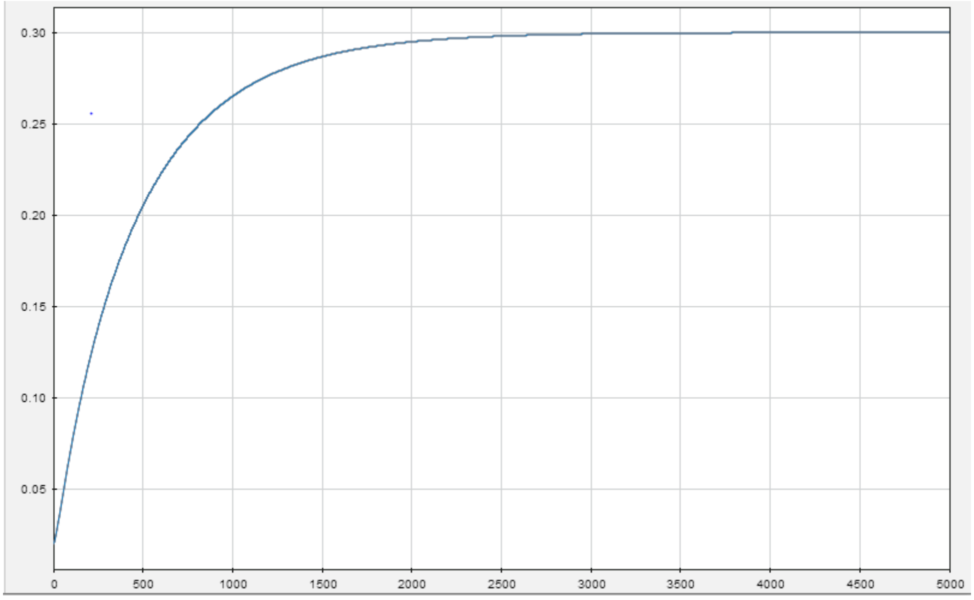

Figure III.8 — Comparison of the level H3 between the dynamic nonlinear model and the TS mode

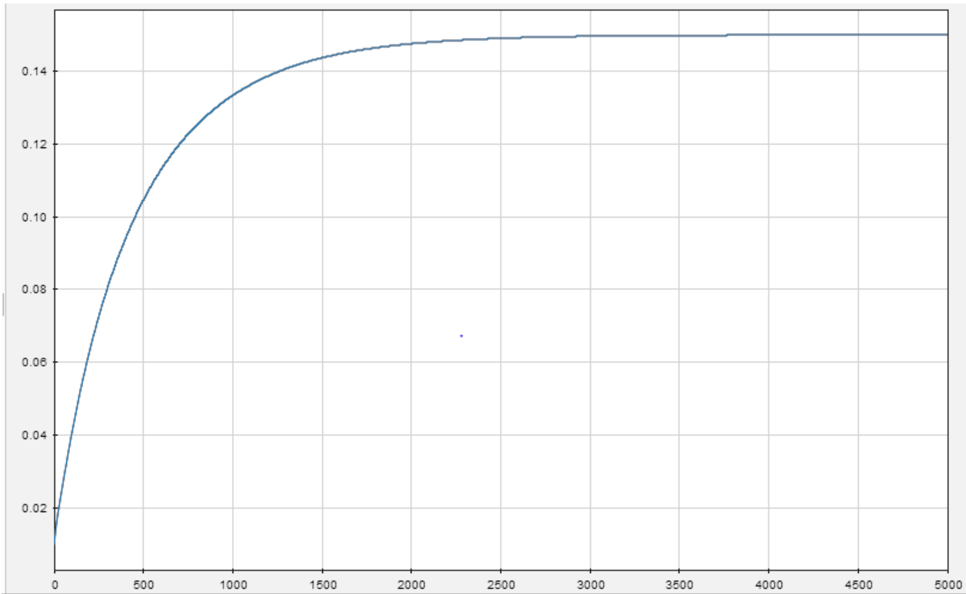

Figure III.7 — Comparison of the level H2 between the dynamic nonlinear model and the TS model

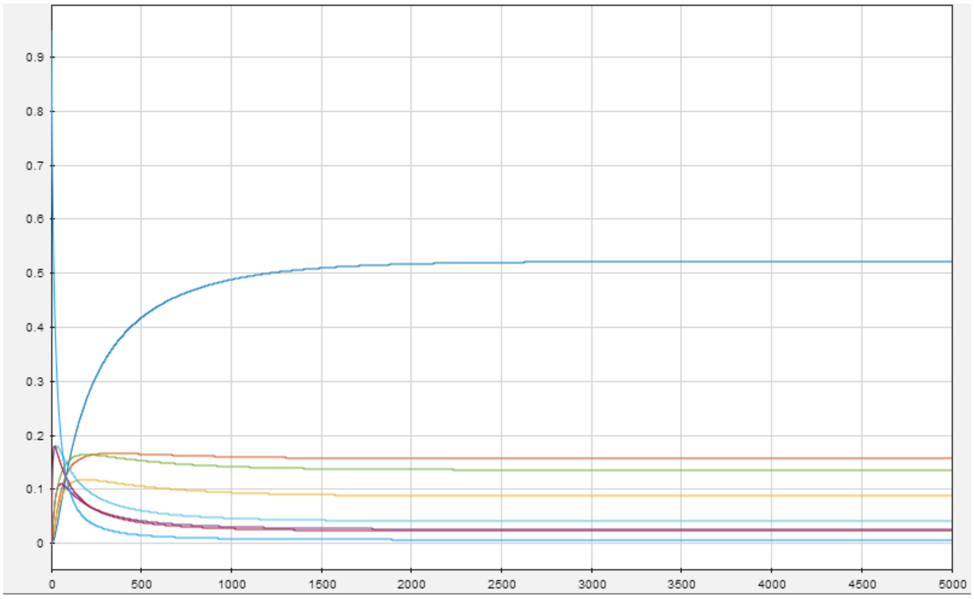

FIGURE III.9 — Membership functions of this system

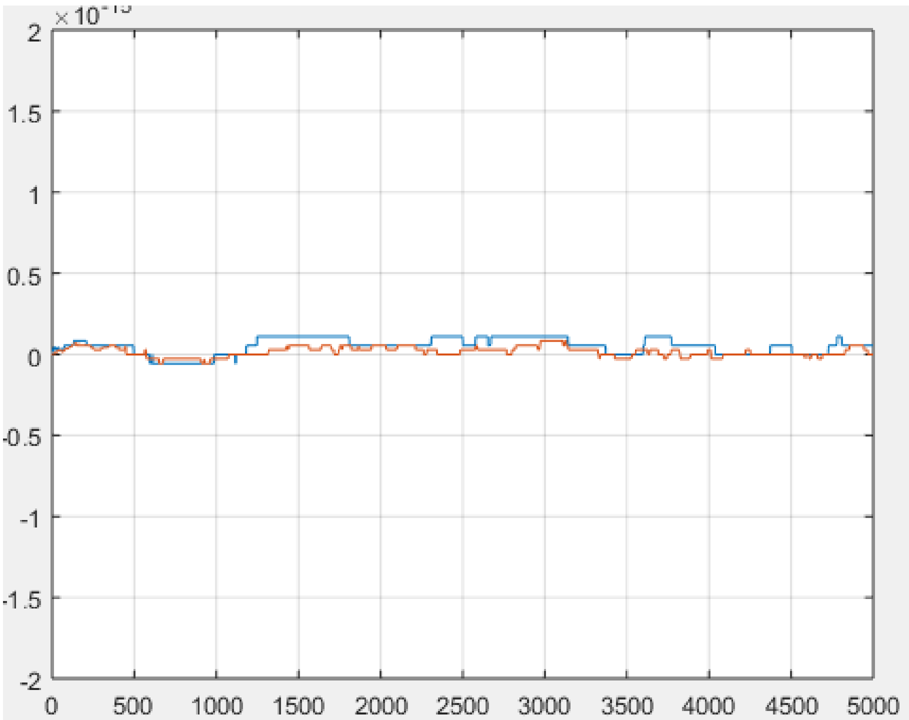

FIGURE III.10 — Difference between the TS and the nonlinear model

III.4.2.2 Second simulation :

Firstly,we will be running the simulation on the following parameters :

— Functioning point : $\begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \end{bmatrix} = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.35 \end{bmatrix}$

— initial conditions : $X_0 = \begin{bmatrix} 0.05 \\ 0.020 \\ 0.035 \end{bmatrix}$

— Simulation step : 0.1 sec .

— Simulation time : [0 ; 5000].

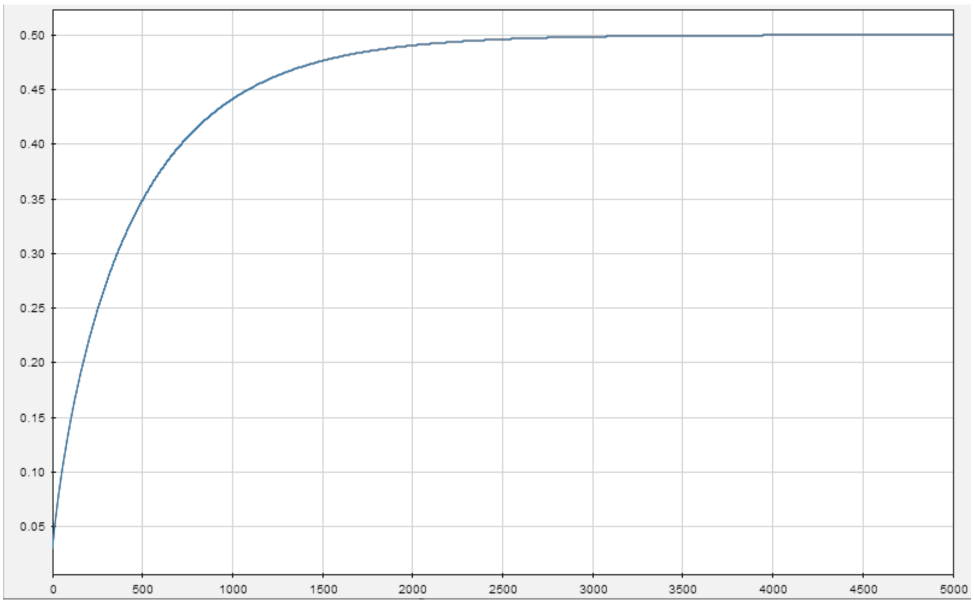

FIGURE III.11 — Comparison of the level H1 between the dynamic nonlinear model and the TS mode

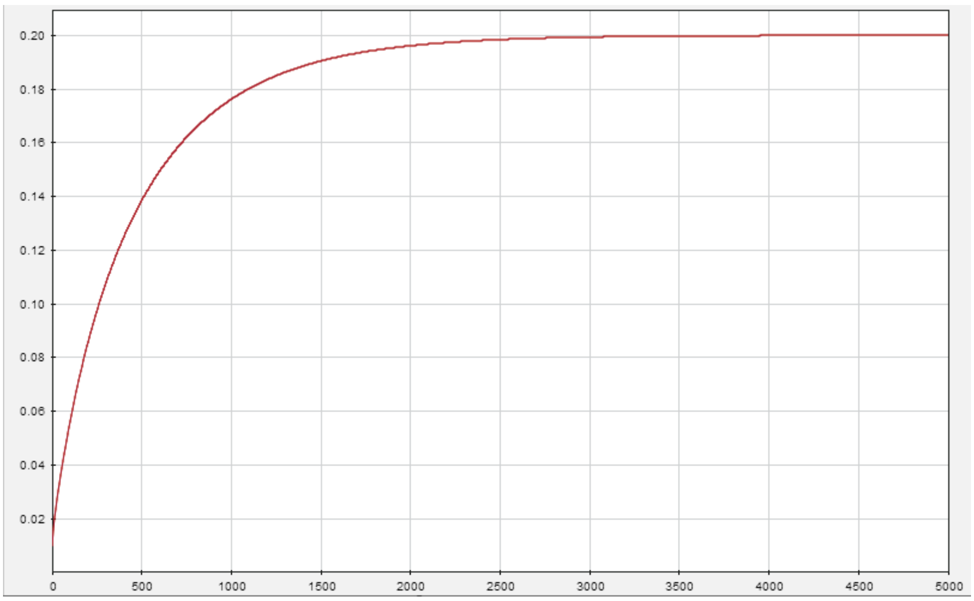

FIGURE III.12 — Comparison of the level H2 between the dynamic nonlinear model and the TS mode

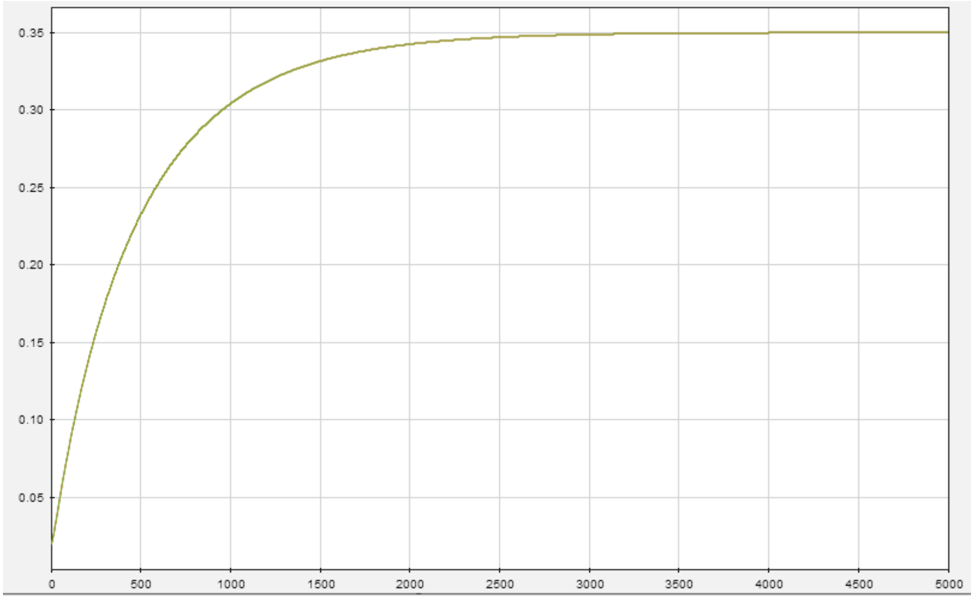

FIGURE III.13 — Comparison of the level H1 between the dynamic nonlinear model and the TS mode

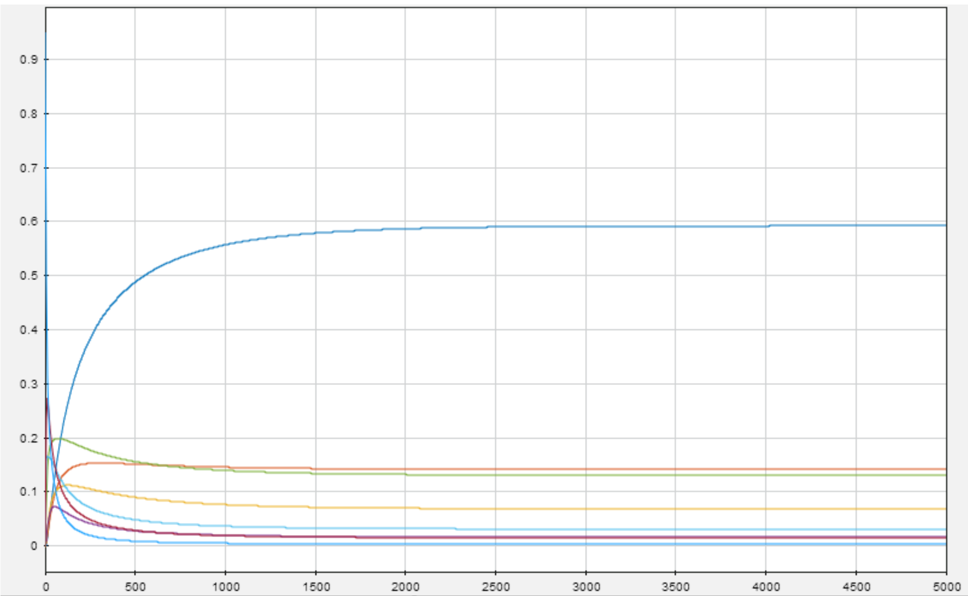

FIGURE III.14 — Membership functions

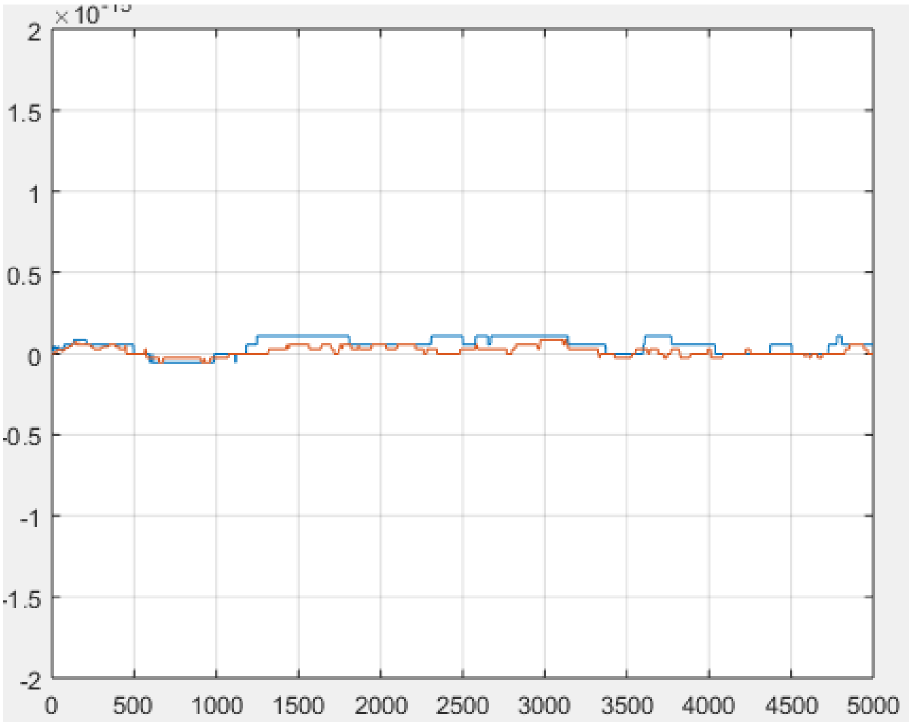

FIGURE III.15 — Difference between the TS and the nonlinear model

III.4.2.3 Results and observations

— First of all,we can clearly see that the results of the simulation are correct and as expected , where $w_i(z) \geq 0$ and $\sum_{i=1}^{m} w_i(z) = 1$.(conditions are satisfied) and each bloc outing is right.

— The error results are $e1 = 1.46e^{-}16, e_3 = 5.44e^{-}17, e_3 = 3.33^{-}17$.

— The responses from both models demonstrate remarkable precision, accurately capturing the system's behavior from the exact initial points we have set, to the functioning points in both times (even for additional operating points that were not mentioned within the scope of this work).This consistent performance underscores the reliability and robustness of this approach using the Matlab Simulink tool in modeling and simulating complex systems with high fidelity.

— The responses of the TS model are identical to those of the nonlinear model, both in the transient and steady-state regimes(permanent regime), with an almost negligible error of $(10^{-}16)$.

III.4.3 Conclusion

In this chapter ,we have successfully applied the Takagi-Sugeno (TS) fuzzy modeling approach to model a complex hydraulic system (three tanks system).where by using the techniques mentioned in the previous chapter we where able to create a TS model that has the same behavior as our hydraulic nonlinear system .
By using the sector non linearity approach we managed to obtain a relatively precise representation of the nonlinear system where the difference between this and the TS model was so insignificant (10^-16) proving that this method is a such powerful tool for accurately representing the dynamic behavior of the system . So as a result we can say that the obtained TS model is valid and we can count on it to further represent and study our hydraulic system.
In the validation and simulation process it is key to highlight the importance of re-calibration of the limits of the membership functions and always making sure that the conditions $w_i(z) \geq 0$ and $\sum_{i=1}^{m} w_i(z) = 1$ are satisfied, in order to get the correct responses that we look for .

Overall,we have manged to obtain a TS model that contains (8) local models(sub models) that has the same behavior / representation of the overall hydraulic system, so now instead of dealing with the complexities of this system we get to work on smaller and more manageable sub models .

Chapter IV

STABILIZATION CONTROL OF THE HYDRAULIC SYSTEM

IV.1 Introduction

In this chapter our main objective is to delve into the analysis of stability and control design for Takagi-Sugeno (TS) models. When it comes to evaluating stability or designing observers and controllers for TS systems, Linear Matrix Inequality (LMI) constraints are widely employed. As such, the chapter begins with a concise overview of LMIs and their advantageous properties. Furthermore, many challenges encountered can be transformed into a multiple-sum co-positivity problem, a well-known issue for which several results are provided. Typically, Lyapunov's direct method is utilized to establish stability and stabilization results for TS models. Generally speaking, for the sake of simplicity and to facilitate expressing problems in an LMI format, the focus is primarily on employing a quadratic Lyapunov function. This approach simplifies the concept of stability to quadratic stability. However, the chapter also presents findings that extend beyond the quadratic framework. The primary objective of this chapter is to highlight the advantages of using Takagi-Sugeno fuzzy models for control applications. To ensure conciseness and readability, the chapter focuses specifically on the stability analysis and state feedback control for particular classes of TS models.

IV.2 Linear Matrix Inequalities fundamentals

IV.2.1 Fundamental notation

Let $F = F^T \in \mathbb{R}^{n\times n}$ be a symmetric matrix. In the sequel, $F > 0$ (resp. $F < 0$) stands for positive (resp. negative)-definiteness, i.e., every eigenvalue of F is strictly positive (resp. negative). The notation $F \geq 0$ (resp. $F \leq 0$) stands for semi-positive (resp. negative), i.e., the eigenvalues can be positive (resp. negative) or zero. Moreover, whenever an expression is written as $F > 0$, it is assumed that the expression is symmetric, i.e., $\hat{F} = F^T > 0$, even if the explicit notation is omitted.
With A, $B \in \mathbb{R}^{n\times n}$ being two symmetric matrices $A > B$ is equivalent to $A - B > 0$.

IV.2.2 Linear Matrix Inequalities

LMIs, or Linear Matrix Inequalities, are matrix inequalities that are either linear or affine in a set of matrix variables. They essentially serve as convex constraints, making them effective in optimizing problems with convex objective functions. This convex nature makes them well-suited for efficiently solving various control problems with convex objectives.This approach has gained significant popularity control engineering in recent years because they can easily solve a wide range of control problems due to their standardized

form and compatibility with existing software.
An LMI has the following form :

$$F(x) = F_0 + x_1F_1 + \cdots + x_nF_n = F_0 + \sum_{i=1}^{n} x_iF_i > 0 \tag{IV.1}$$

where $x \in R^m$ is the vector of decision variables and $F_0, F_1, \ldots, F_n$ are given constant symmetric real matrices, i.e., $F_i = F_i^T,\ i = 0, \ldots, m$. The inequality symbol in the equation means $F(x)$ is positive definite, i.e., $u^T F(x)u > 0$ for all nonzero $u \in R^n$. This matrix inequality is linear in the variables x_i.
The set of solutions of the LMI or the so-called feasibility set, denoted by $S = \{x|x \in \mathbb{R}^m, F(x) > 0\}$ is a convex subset of $\mathbb{R}^m$. Finding a solution to (IV.1) is a convex optimization problem avoiding local minima and guaranteeing finite feasibility tests. When no solution exists, the problem is said to be infeasible. The following well-known convex or quasi-convex optimization problems are relevant for the analysis and the synthesis of control systems [12].
The task of finding a solution $x \in \mathbb{R}^m$ for the LMI system (IV.1), or determining that there is no solution (ineffability) , is known as the feasibility problem (FP). This problem can be rephrased as minimizing the convex function $f : x \to \lambda_{\min}(F(x))$, where $\lambda_{\min}$ represents the smallest eigenvalue, and then determining whether the resulting solution is positive (strictly feasible), zero (feasible), or negative (infeasible).
On the other hand, minimizing a linear combination of decision variables $b^T x$ subject to (IV.1) constitutes the eigenvalue problem (EVP), also referred to as an LMI optimization problem.
Furthermore, the task of minimizing the eigenvalues of a pair of matrices that are affinely dependent on a variable, subject to a set of LMI constraints, or determining the problem's infeasibility, can be expressed as solving the problem :

$$\text{minimize} \quad \lambda \text{ subject to :} \begin{cases} \lambda B(x) - A(x) > 0 \\ B(x) > 0 \\ C(x) > 0 \end{cases}$$

where $A(\boldsymbol{x}), B(\boldsymbol{x})$ and $C(\boldsymbol{x})$ are symmetric and affine with respect to $\boldsymbol{x}$,is called a generalized eigenvalue problem (GEVP).

IV.2.3 Proprieties

In control problems, LMI constraints don't typically appear spontaneously. However, leveraging optimization solutions allows us to transform control problems into LMI expressions. This process focuses on the inherent properties of LMIs. Below are some of these properties :

Property 1 :(Congruence)Given a matrix $P = P^T$and a full column rank matrix$Q$ it holds that

$$P > 0 \Rightarrow QPQ^T > 0$$

Property 2 : (Schur complement) Consider a matrix $M = M^T = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}$, with M_{11} and M_{22} being square matrices. Then

$$M < 0 \Leftrightarrow \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \Leftrightarrow \begin{cases} M_{22} < 0 \\ M_{11} - M_{12} M_{22}^{-1} M_{12}^T < 0 \end{cases}$$

Property 3 : (S-procedure) Consider matrices $F_i = F_i^T \in \mathbb{R}^{n \times n}, x \in \mathbb{R}^n$, such that $x^T F_{\text{i}} x \geq 0, i = 1, \ldots, p$, and the quadratic inequality condition

$$x^T F_0 x > 0$$

$x \neq 0$. A sufficient condition for (32) to hold is : there exist $\tau_i \geq 0, i = 1, \ldots, p$, such that $F_0 - \sum_{i=1}^{p} \tau_i F_i > 0$.

IV.3 Stability Analysis of TS Systems

In this section we delve into methods that are used the stability analysis of TS fuzzy systems. IN other words, we will present a stability analysis and stabilization of TakagiSugeno models using LMI constraints. The TS model used is :

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{m} w_i(z) \left(A_i x + B_i u \right) \\ y &= \sum_{i=1}^{m} w_i(z) C_i x \end{aligned} \tag{IV.2}$$

IV.3.1 Quadratic Stability

The stability of TS models is investigated using Lyapunov's direct method. The commonly employed Lyapunov function is a quadratic one, denoted as

$$V(x) = x^T P x \tag{IV.3}$$

where $P = P^T > 0$. When utilizing this quadratic Lyapunov function, the concept is referred to as "quadratic stability." It is important to note that while quadratic stability implies stability, the reverse is not necessarily true. Consequently, the conditions obtained using the Lyapunov function are only sufficient. In other words, if the LMI conditions fail, no direct conclusion can be drawn regarding the stability or instability of the TS model. For the unforced ($u = 0$) TS model, quadratic stability is achieved if the Lyapunov function decreases and converges to zero as t approaches infinity for all trajectories x(t). The derivative of $V(x)$along the trajectories of the unforced model (IV.3) is given by :

$$\begin{aligned}\dot{V} &= \left(\sum_{i=1}^{m} w_i(z) A_i x\right)^T P x + x^T P \left(\sum_{i=1}^{m} w_i(z) A_i x\right) \\ &= \sum_{i=1}^{m} w_i(z) x^T \left(A_i^T P + P A_i\right) x\end{aligned}$$

Remembering that $w_\mathrm{i}(\boldsymbol{z}) \geq 0, i = 1, 2, \ldots, m$ the following theorem is straight forwardly obtained [13].

Theorem 1 : The fuzzy model $\dot{x} = \sum_{i=1}^{m} w_i(z)(A_i x =$ is globally asymptotically stable if there exists a common positive definite matrix $P = P^T > 0$, such that :

$$A_i^T P + P A_i < 0 \qquad i = 1, ..., r$$

The condition states that for the continuous fuzzy model to be globally asymptotically stable, there must exist a common positive definite matrix P, which satisfies these criteria(inequality).

Property : If there exist positive definite matrices $R_i = R_i^T > 0, i = 1, 2, \ldots, m$ such that :

$$\sum_{i=1}^{m} \left(A_i^T R_i + R_i A_i\right) > 0$$

then there is no matrix $P = P^T > 0$ such that the theorem conditions hold .

IV.4 State Feedback Stabilization

To achieve stabilization of a Takagi-Sugeno (TS) system using state feedback, several control laws can be employed, one of which is the linear feedback $u = -Kx$. A more general solution is the Parallel Distributed Compensation (PDC) scheme. The PDC is composed of linear state feedback controllers that are blended together using the nonlinear membership functions ω_i associated with each mode or regime of the TS model.such as

$$u = -\sum_{i=1}^{m} w_i(\boldsymbol{z}) K_i \boldsymbol{x}$$

And by introducing this in the TS model (IV.2) we get :

$$\begin{aligned}
\dot{x} &= \sum_{i=1}^{m} w_i(z) \left(A_i - B_i \sum_{j=1}^{m} w_j(z) K_j \right) x \\
&= \sum_{i=1}^{m} w_i(z) \left(\underbrace{\sum_{j=1}^{m} w_j(z)}_{=1} A_i - B_i \sum_{j=1}^{m} w_j(z) K_j \right) x
\end{aligned} \tag{IV.4}$$

and finally, the closed loop is composed of m^2 linear models

$$\dot{x} = \sum_{i=1}^{m} \sum_{j=1}^{m} w_i(z) w_j(z) \left(A_i - B_i K_j \right) \boldsymbol{x} \tag{IV.5}$$

In other words, the PDC approach involves designing multiple linear state feedback controllers, each tailored to a specific operating region or mode of the TS system. These linear controllers are then combined in parallel, with their outputs weighted by the corresponding nonlinear membership functions that describe the system's current mode of operation. This blending of linear controllers, based on the nonlinear membership functions, results in an overall nonlinear control law that can effectively stabilize the TS system across its entire operating range. Compared to the simpler linear feedback law, the PDC scheme provides a more comprehensive and flexible solution [14].
Furthermore, going withe quadratic stability as discussed earlier, pertains to examining the derivative of the Lyapunov function along the trajectories of system (IV.5)

$$\dot{V} = \sum_{i=1}^{m} \sum_{j=1}^{m} w_i(z) w_j(z)\, x^T \left(\left(A_i - B_i K_j \right)^T P + P \left(A_i - B_i K_j \right) \right) x$$

Where $\dot{V} < 0$ is ensured if the double sum negativity problem (3.3) is satisfied, in which this case can be written as

$$\sum_{i=1}^{m}\sum_{j=1}^{m} w_i(z)w_j(z)\left(A_i^T P + PA_i - PB_iK_j - K_j^T B_i^T P\right) < 0$$

This analysis helps determine the system's stability characteristics, such as whether it tends towards an equilibrium point or exhibits oscillatory behavior around it. Note also that due to the quantity PB_iK_j, this expression is not an LMI.To express it with LMI conditions, the following change of variables can be performed : $\dot{X} = P^{-1}, M_i = K_iX, i = 1, 2, \ldots, m$ and with the property of congruence with full rank matrix X is equivalent to

$$\sum_{i=1}^{m}\sum_{j=1}^{m} w_i(z)w_j(z)\left(XA_i^T + A_iX - B_iM_j - M_j^T B_i^T\right) < 0$$

The conclusion results is presented as in the following theorem.
Theorem : The continuous TS model with the PDC control law (**??**) is globally asymptotically stable if there exist matrix $P = P^T > 0$ that satisfies the condition :

$$(A_i - B_iK_j)^T.P + P.(A_i - B_iK_j) < 0 \quad , j = 1, ..., r$$

IV.5 Stabilisation of the three tank system

Now that we have successfully established the Takagi-Sugeno (TS) model that accurately represents the three tank system, with a resulting TS model comprises 8 linear subsystems, each capturing the system's dynamics within a specific operating region of the overall operational domain.Now pur primary objective in this synthesis is to conduct a comprehensive stability analysis of these local linear subsystems and ensure the global stability of the overall TS fuzzy model. Achieving global stability is crucial for guaranteeing the reliable and robust performance of the three tank system across varying operating conditions. To this end, we will investigate the stability properties of TS fuzzy models.
For the stabilization of the system, we will employ the Parallel Distributed Compensation (PDC) control strategy. In summary this approach involves designing and integrating multiple linear controllers, each tailored to a specific operating region represented by a local linear subsystem within the TS fuzzy model. The outputs of these local controllers are then blended using the nonlinear membership functions of the TS model, resulting in

a comprehensive nonlinear control law capable of stabilizing the overall system across its entire operational domain.

IV.5.1 Achieving stability of the three tank system

As previously established our model is in the form :

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{m} w_i(z)\left(A_i x + Bu\right) \quad , i = 1, ..., 8 \\ y &= \sum_{i=1}^{m} w_i(z) C_i x \end{aligned} \tag{IV.6}$$

For the stabilization of the system, we will employ a nonlinear state feedback control approach with constant gains K, weighted by the same membership functions used in the Takagi-Sugeno (TS) model. The control law takes the following form :

$$u(t) = -\sum_{i=1}^{r} w_i\left(z(t)\right) K_i x(t)) \tag{IV.7}$$

By replacing $u(t)$ in (IV.6) :

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} w_i(z(t))(A_i x(t) + B_i\left(-\sum_{j=1}^{r}{}_{j}\left(z(t)\right) K_j x(t)\right)) \\ \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_j\left(z(t)\right) w_i(z(t))(A_i - B_i K_j)x(t) \end{aligned} \tag{IV.8}$$

According to the preceding theorems, for our system to be stable, the candidate Lyapunov function $V(x(t))$ must be positive definite, This means that the function $V(x(t))$ is positive for all non-zero values of the state vector $x(t)$,And derivative $\dot{V}(x(t))$, must be negative definite.
So :

$$\mathrm{V}\big(\mathrm{x(t)}\big) = \mathrm{x}^{\mathrm{T}}\mathrm{(t).P.x(t)} \quad \text{Where} \quad \mathrm{P} = \mathrm{P}^{T} > 0$$

$$\dot{\mathrm{V}}\big(\mathrm{x(t)}\big) = \dot{\mathrm{x}}^{\mathrm{T}}\mathrm{(t).P.x(t)} + \mathrm{x}^{\mathrm{T}}\mathrm{(t).P.}\dot{\mathrm{x}}\mathrm{(t)}$$

And by replacing x with (**??**) :

$$\begin{aligned} \dot{\mathrm{v}}(\mathrm{x(t)}) = &\left(\sum_{i=1}^{r}\sum_{j=1}^{r} w_j\left(z(t)\right) w_i(z(t))\left(A_i - B_i K_j\right) x(t)\right)^{\mathrm{T}} \mathrm{(t).P.x(t)} \\ &+ \mathrm{x}^{\mathrm{T}}\mathrm{(t).P.}\left(\sum_{i=1}^{r}\sum_{j=1}^{r} w_j\left(z(t)\right) w_i(z(t))(A_i - B_i K_j)x(t)\right) \end{aligned}$$

$$\dot{\mathrm{V}}\big(x(\mathrm{t})\big) = x^T(\mathrm{t}).\left(\sum_{i=1}^{r}\sum_{j=1}^{r} w_j\big(z(t)\big)w_i\big(z(t)\big)\left[\big(A_i - B_iK_j\big)^T\mathrm{P} + \mathrm{P}\,(A_i - B_iK_j)\,\right]\right).\mathrm{x(t)}$$

Taking into consideration that : $\sum_{i=1}^{r} w_i\big(z(t)\big)\text{And}\sum_{j=1}^{r} w_j\big(z(t)\big) = 1$

$$\begin{aligned}
&(\mathrm{A}_i - B_iK_j)^T\mathrm{P} + \mathrm{P}\,(\mathrm{A}_i - B_iK_j) < 0\\
&\mathrm{A_i}^T\mathrm{P} - B_i^T\,K_j^T\mathrm{P} + \mathrm{P}\,\mathrm{A_i} - P\,B_iK_j < 0\\
&\mathrm{A_i}^T\mathrm{P} + \mathrm{P}\,\mathrm{A_i} - (B_i^T\,K_j^T\mathrm{P} + P\,B_iK_j) < 0
\end{aligned} \qquad \text{(IV.9)}$$

By multiplying from both sides with P^{-1} we get :

$$\mathrm{P}^{-1}\mathrm{A}_i^T + \mathrm{A}_i\mathrm{P}^{-1} - (\mathrm{P}^{-1}B_i^TK_j^T + B_iK_j\mathrm{P}^{-1}) < 0$$

Notice that this is expression is not an LMI, where P and K_j are unknown variables so this expression is not linear (which makes it not an lmi),to solve this issue we will preform a variables change where $Q{=}P^{-1}$ *and* $X_j = K_j\mathrm{P}^{-1} = K_j\mathrm{Q}$ giving :

$$Q\mathrm{A}_i^T + \mathrm{A}_i\mathrm{Q} - (X_j^TB_i^T + B_iX_j) < 0 \qquad \text{wherej} = 1, ..., 8.$$

Our TS system contains 8 sub models (sub systems) meaning $j = i = 1, ..., 8$. This gives us a feasibility problem of a linear matrix inequalities (LMIs), which has $8^2 + 1 = 65$ constraints that need to be solved constructed as follow :

$$\begin{aligned}
&\mathrm{Q} > 0\\
&Q\mathrm{A}_1^T + \mathrm{A}_1\mathrm{Q} - (X_1^TB^T + BX_1) < 0\\
&Q\mathrm{A}_2^T + \mathrm{A}_2\mathrm{Q} - (X_1^TB^T + BX_1) < 0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&Q\mathrm{A}_8^T + \mathrm{A}_8\mathrm{Q} - (X_1^TB^T + BX_1) < 0\\
&Q\mathrm{A}_1^T + \mathrm{A}_1\mathrm{Q} - (X_2^TB^T + BX_2) < 0\\
&Q\mathrm{A}_2^T + \mathrm{A}_2\mathrm{Q} - (X_2^TB^T + BX_2) < 0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&Q\mathrm{A}_8^T + \mathrm{A}_8\mathrm{Q} - (X_2^TB^T + BX_2) < 0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&Q\mathrm{A}_7^T + \mathrm{A}_7\mathrm{Q} - (X_8^TB^T + BX_8) < 0\\
&Q\mathrm{A}_8^T + \mathrm{A}_8\mathrm{Q} - (X_8^TB^T + BX_8) < 0
\end{aligned}$$

In order for the system to be stable, there must exist a common matrix Q that satisfies all of these 65 constraints.

IV.6 Solving LMI Using MATLAB Toolbox

The LMI (Linear Matrix Inequality) toolbox in MATLAB offers a set of useful functions to solve LMI problems. Some of these functions are discussed here, along with sample codes.

Step 1 : Initialization

To begin, initialize the LMI description by using the command setlmis([]). Note that this function does not require any parameters.

Step 2 : Defining the Decision Variables

Next, it is necessary to define the decision variables, which are the unknown variables of the LMI problem. Consider the example LMI $C^T XC < 0$, where C is a constant matrix and X is the matrix of decision variables. The decision variables are defined using the lmivar function, which has the following syntax :

$$X = \text{lmivar}(\text{type}, \text{structure}).$$

This command allows us to define several forms of decision matrices such as symmetrical matrices, rectangular matrices or matrices of other type. Depending on the selected matrix type, the structure contains different information. Thus, first we define the type and then define the structure which depends on the type.

Step 3 : Constructing the LMI Constraints

After defining the decision variables, we can now construct the LMI constraints using *lmiterm* which takes the LMI expression as an argument.

$$\text{lmiterm}(\text{termID}, \text{A}, \text{B}, \text{flag}).$$

Step 4 : Solving the LMI Problem

Finally, you can solve the LMI problem by invoking the solver function *getlmisolvers* and selecting an appropriate solver based on the problem characteristics and requirements.

This LMI toolbox in MATLAB provides a powerful and flexible framework for defining and solving LMI problems [15].

IV.6.1 Application

By using the Matlab toolbox for solving LMIs we managed to generate the LMI constraint needed in our system by defining the decision variables $Q{=}P^{-1}$ *and* $X_j = K_j \mathrm{P}^{-1} = K_j \mathrm{Q}$ and introducing all the 65 constraints that where established earlier as (Figure IV.1) shows :

```
 Solver for LMI feasibility problems L(x) < R(x)
    This solver minimizes  t  subject to  L(x) < R(x) + t*I
    The best value of t should be negative for feasibility

 Iteration   :    Best value of t so far

* switching to QR
     1                        -0.028886

 Result:  best value of t:    -0.028886
          f-radius saturation:  0.004% of R =  1.00e+09


PS =

    3.9474   -0.0022   -2.5480
   -0.0022    3.4599    0.0365
   -2.5480    0.0365   13.4206
```

Figure IV.1 — Matlab LMI solver results

Now that the conditions are satisfied We can now identify the PDC gains (K_j) of our TS model , where :

$$X_j = K_j \mathrm{P}^{-1} = K_j \mathrm{Q} \quad \text{So} \quad K_j = X_j \mathrm{P} = X_j \mathrm{Q}$$

Giving the PDC gains of our TS model as :

$$\mathrm{K}_1 = (10^3).\begin{bmatrix} 0.0053 & -3.8810 & 0.0116 \\ 3.8767 & -0.0053 & 0.7360 \end{bmatrix} \quad , \quad \mathrm{K}_2 = (10^3).\begin{bmatrix} 0.0024 & -1.7488 & 0.0052 \\ 1.7469 & -0.0024 & 0.3317 \end{bmatrix}$$

$$\mathrm{K}_3 = (10^3).\begin{bmatrix} -0.0039 & 2.8489 & -0.0085 \\ -2.8457 & 0.0039 & -0.5403 \end{bmatrix} \quad , \quad \mathrm{K}_4 = (10^3).\begin{bmatrix} 0.0022 & -1.5892 & 0.0047 \\ 1.5874 & -0.0022 & 0.3014 \end{bmatrix}$$

$$K_5 = (10^3).\begin{bmatrix} 0.0222 & -9.1868 & 0.0293 \\ 9.1766 & -0.0037 & 1.7426 \end{bmatrix} \quad , \quad K_6 = (10^3).\begin{bmatrix} 0.0060 & -4.3765 & 0.0130 \\ 4.3716 & -0.0060 & 0.8300 \end{bmatrix}$$

$$K_7 = (10^3).\begin{bmatrix} 0.0030 & -2.1927 & 0.0065 \\ 2.1902 & -0.0030 & 0.4158 \end{bmatrix} \quad , \quad K_8 = (10^3).\begin{bmatrix} 0.0053 & -3.8631 & 0.0115 \\ 3.8588 & -0.0053 & 0.7326 \end{bmatrix}$$

IV.6.2 Results and interpretations

The control matrices of our closed-loop system are presented as $Ai_i = A_i - BK_i$, After verification and calculation using Matlab, We have found that the eigenvalues of all the control matrices are all negative.

Finally we determined the matrix P= $\begin{bmatrix} 3.9474 & -0.0022 & -2.5480 \\ -0.0022 & 3.4599 & 0.0365 \\ -2.5480 & 0.0365 & 13.4206 \end{bmatrix}$
such that the eigenvalues of P are strictly positive (P is symmetric and positively defined). Therefore, we get to finally say that our global system is asymptotically stable in the closed-loop. .
The solution of our LMI problem that was obtained using the Matlab toolbox is way too big , where notice that the PDC gains value are too high (10^3), which resulted in generating a substantial command u that where not acceptable in our system and matter effect the value of k was so big to the point where even the simulation using Matlab failed . Considering that the value of the gains K_i is the main factor in the value of the command u ($u = -wkx$) we need to find another value of the decision variables in a way that we can specify them to be smaller than the previous ones and within an acceptable range , to do that we introduce the LMI region.

IV.6.3 LMI region

The LMI region is a concept used in control theory and robust control analysis. It represents a set of matrices that satisfy specific linear matrix inequalities. If a matrix K belongs to this LMI region $S(\alpha, r, \theta)$, it implies that K satisfies certain stability and performance criteria.in our case it for stability of the nonlinear system .By checking if a matrix A lies within the LMI region, one can ensure that the system meets certain stability and performance requirements [16].

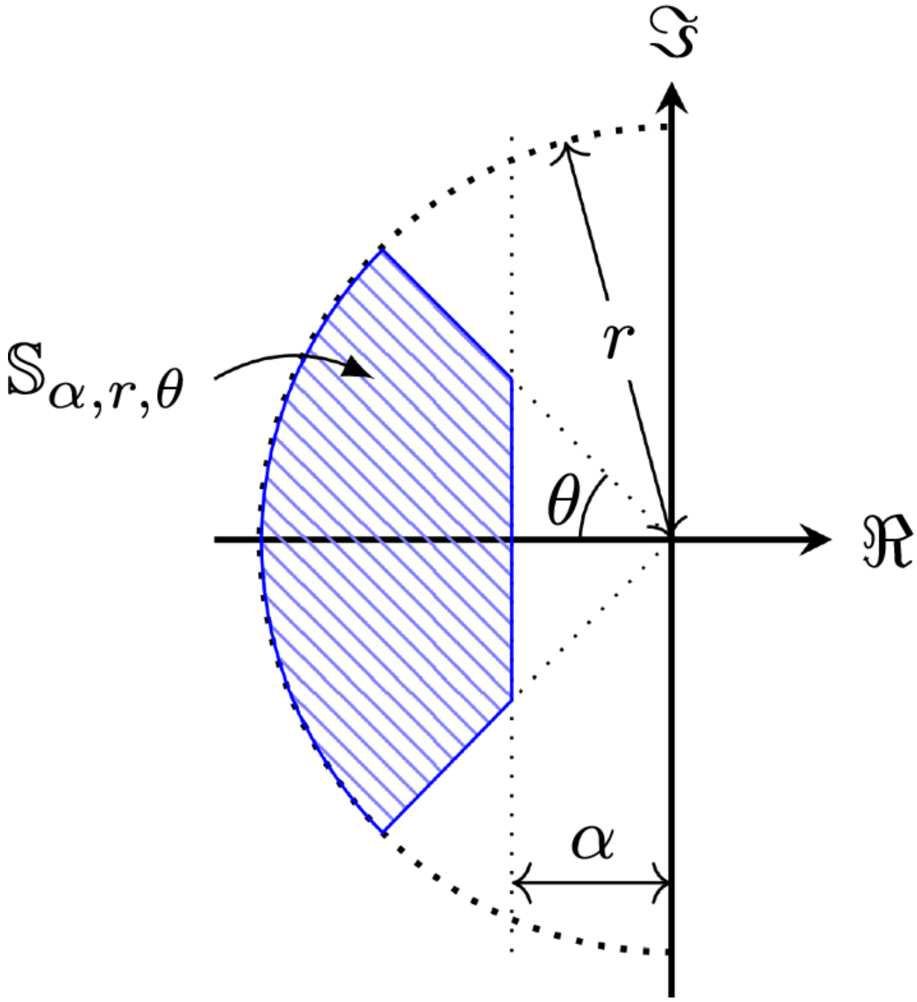

FIGURE IV.2 — LMI region

The image shows the geometric representation of the LMI region, with the parameters α, randθ defining the shape and boundaries of the shaded area. The equations (IV.10), (IV.11), and (IV.12) provided below , define the specific linear matrix inequalities that a matrix $\hat{A}_{ij}$ must satisfy to be considered within the LMI region S$(\alpha, r, \theta).2\alpha P + \hat{A}_{ij}P + P\hat{A}_{ij}^{\top} < 0,$ (IV.10)

$$\begin{bmatrix} -rP & \hat{A}_{ij}P \\ P\hat{A}_{ij}^{\top} & -rP \end{bmatrix} < 0, \tag{IV.11}$$

$$\begin{bmatrix} \left(\hat{A}_{ij}P + P\hat{A}_{ij}^{\top}\right)\sin\theta & \left(\hat{A}_{ij}P - P\hat{A}_{ij}^{\top}\right)\cos\theta \\ \left(P\hat{A}_{ij}^{\top} - \hat{A}_{ij}P\right)\cos\theta & \left(\hat{A}_{ij}P + P\hat{A}_{ij}^{\top}\right)\sin\theta \end{bmatrix} < 0. \tag{IV.12}$$

By applying this to our system,where the matrices is actually the control matrices of our closed loop system such like :

$$\hat{A_{ij}} = Ai_i = (A_i - BK_i) \tag{IV.13}$$

So will have a combination of 64 matrices which means 64 constraints for each of the parameters $\alpha, r and \theta$ that defines the LMI region we want our solutions to be in .so in total we will have 256 constraint . presented as :

$$\begin{aligned}
&\mathrm{Q} > 0\\
&Q\mathrm{A}_1^T + \mathrm{A}_1\mathrm{Q} - (X_1^T B^T + BX_1) < 0\\
&2\alpha P + \hat{A_{11}} P + P\hat{A_{11}}^{\mathsf{T}} < 0\\
&\begin{bmatrix} -rP & \hat{A_{11}}P \\ P\hat{A_{11}}^{\mathsf{T}} & -rP \end{bmatrix} < 0\\
&\begin{bmatrix} \left(\hat{A_{11}}P + P\hat{A_{11}}^{\mathsf{T}}\right)\sin\theta & \left(\hat{A_{11}}P - P\hat{A_{11}}^{\mathsf{T}}\right)\cos\theta \\ \left(P\hat{A_{11}}^{\mathsf{T}}\hat{A_{11}}P\right)\cos\theta & \left(\hat{A_{11}}P + P\hat{A_{11}}^{\mathsf{T}}\right)\sin\theta \end{bmatrix} < 0
\end{aligned} \tag{IV.14}$$

$$\begin{aligned}
&Q\mathrm{A}_1^T + \mathrm{A}_1\mathrm{Q} - (X_2^T B^T + BX_2) < 0\\
&2\alpha P + \hat{A_{11}} P + P\hat{A_{11}}^{\mathsf{T}} < 0\\
&\begin{bmatrix} -rP & \hat{A_{12}}P \\ P\hat{A_{12}}^{\mathsf{T}} & -rP \end{bmatrix} < 0\\
&\begin{bmatrix} \left(\hat{A_{12}}P + P\hat{A_{12}}^{\mathsf{T}}\right)\sin\theta & \left(\hat{A_{12}}P - P\hat{A_{12}}^{\mathsf{T}}\right)\cos\theta \\ \left(P\hat{A_{12}}^{\mathsf{T}}\hat{A_{12}}P\right)\cos\theta & \left(\hat{A_{12}}P + P\hat{A_{12}}^{\mathsf{T}}\right)\sin\theta \end{bmatrix} < 0
\end{aligned}$$

Until the 265 constraint :

$$\begin{aligned}
&QA_8^T + \mathrm{A}_8\mathrm{Q} - (X_8^T B^T + BX_8) < 0\\
&2\alpha P + \hat{A_{88}} P + P\hat{A_{88}}^{\mathsf{T}} < 0\\
&\begin{bmatrix} -rP & \hat{A_{88}}P \\ P\hat{A_{88}}^{\mathsf{T}} & -rP \end{bmatrix} < 0\\
&\begin{bmatrix} \left(\hat{A_{88}}P + P\hat{A_{88}}^{\mathsf{T}}\right)\sin\theta & \left(\hat{A_{88}}P - P\hat{A_{88}}^{\mathsf{T}}\right)\cos\theta \\ \left(P\hat{A_{88}}^{\mathsf{T}}\hat{A_{88}}P\right)\cos\theta & \left(\hat{A_{88}}P + P\hat{A_{88}}^{\mathsf{T}}\right)\sin\theta \end{bmatrix} < 0
\end{aligned} \tag{IV.15}$$

Finally,by reintroducing all the 256 constraints that where established earlier to Matlab LMI toolbox with the parameters as ($\alpha = 0.0015; r = 0.01; \theta = 20$) we get the following results : Giving the PDC gains of our TS model as :

$$K_1 = (10^{-3}).\begin{bmatrix} 0.0597 & 0.0193 & -0.0285 \\ -0.0250 & 0.0689 & 0.1775 \end{bmatrix}$$

$$K_2 = K_3 = (10^{-3}).\begin{bmatrix} 0.0595 & 0.0190 & -0.0289 \\ -0.0250 & 0.0686 & 0.1774 \end{bmatrix}$$

$$K_4 = K_5 = K_6 = K_7 = (10^{-3}).\begin{bmatrix} 0.0597 & 0.0193 & -0.0285 \\ -0.0250 & 0.0689 & 0.1775 \end{bmatrix}$$

$$K_8 = (10^{-3}).\begin{bmatrix} 0.0614 & 0.0359 & -0.0604 \\ -0.0467 & -0.0080 & 0.2296 \end{bmatrix}$$

With the eigenvalues :

$$\lambda_1 = \begin{cases} -0.0078 + 0.0042i \\ -0.0078 - 0.0042i \\ -0.0060 + 0.0000i \end{cases}$$

$$\lambda_8 = \begin{cases} -0.0194 + 0.0020i \\ -0.0194 - 0.0020i \\ -0.0109 + 0.0000i \end{cases}$$

All the eigenvalues of the gains K are strictly negative.
As planed, all the results of our LMI problem are within the LMI region that we specified, and the values of the PDC gains are much smaller than before providing (generating) a much more acceptable command u($u_1, u_2 =< 10^-4$)

IV.7 Simulation

In this part we simulate the nonlinear system in state feedback , in other words we will be simulating system (IV.5) using Matlab simulink, Since we are dealing with stabilisation of this system the outputs will converge from initial conditions to zero when achieving stability ,only that now to avoid having a zero in the outings of our system will have to change the equilibrium point to a new point that is different than zero, this is achieved by adding the corresponding command to the already existing command that we have as :

$$u = -\sum_{i=1}^{m} w_i(\boldsymbol{z}) K_i \boldsymbol{x} + u_2$$

So in these functioning point :

$$\begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.020 \\ 0.035 \end{bmatrix}$$

The corresponding commends are :

$$\begin{bmatrix} u_{12} \\ u_{22} \end{bmatrix} = (10^{-}4) \begin{bmatrix} 0.1220 \\ 0.0659 \end{bmatrix}$$

with the following initial condition : $X_0 = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.35 \end{bmatrix}$

The overall new system (IV.5) is simulated as shown in (IV.3).

Under these following simulation parameters :

-Simulation step : 0.1 sec .

-Simulation time : [0 ; 5000].

We can finally get the results of h1,h2 and h3, as well as the command in stat feedback u1 and u2 as the following figures shows .

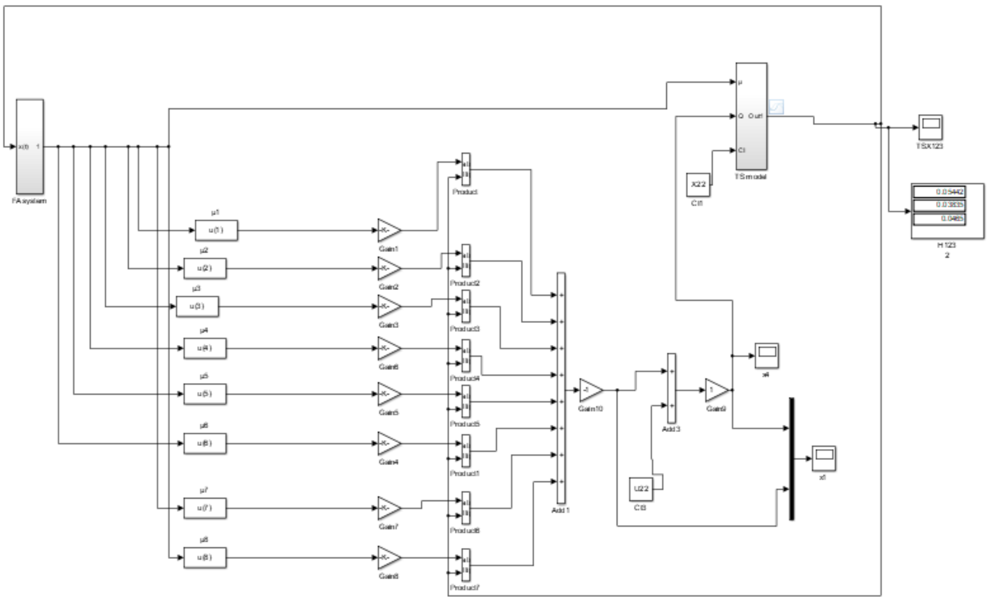

Figure IV.3 — Overall TS nonlinear model in state feedback stabilisation

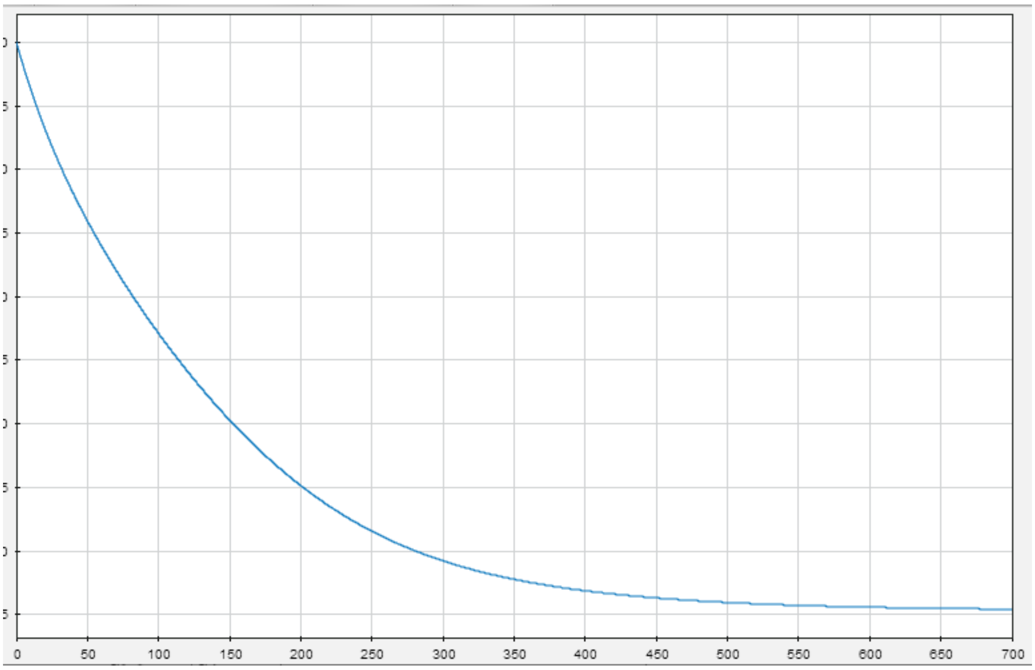

Figure IV.4 — The level h1 of the nonlinear system

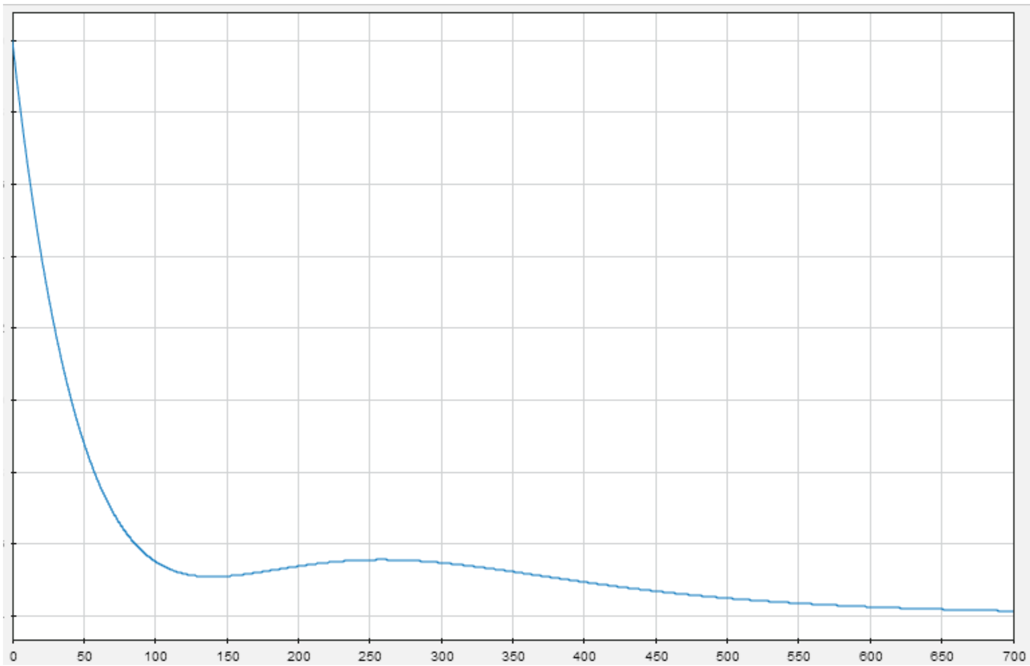

Figure IV.5 — The level h2 of the nonlinear system

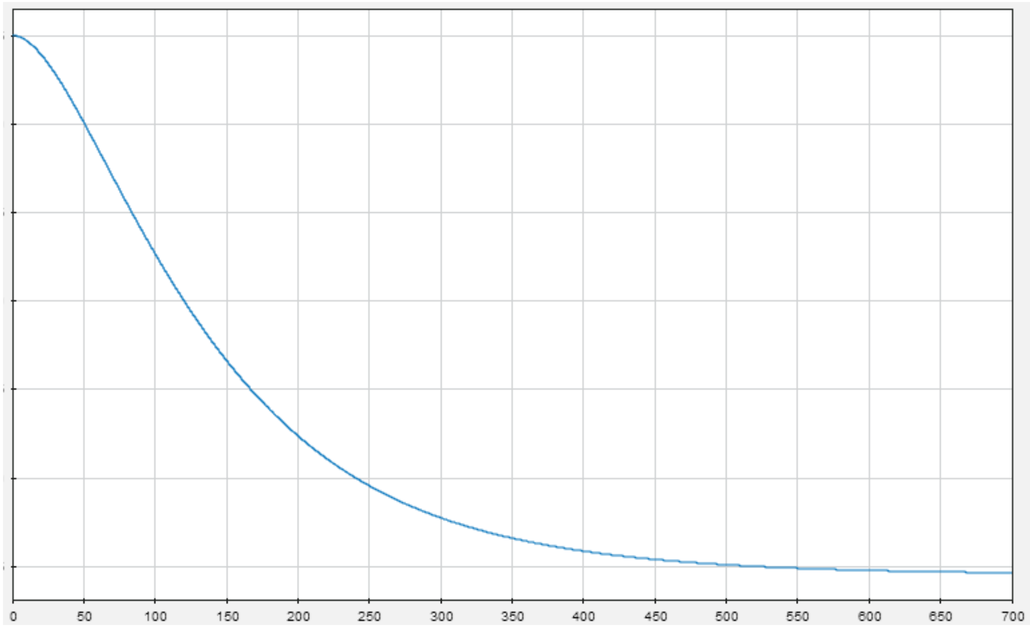

FIGURE IV.6 — The level h3 of the nonlinear system

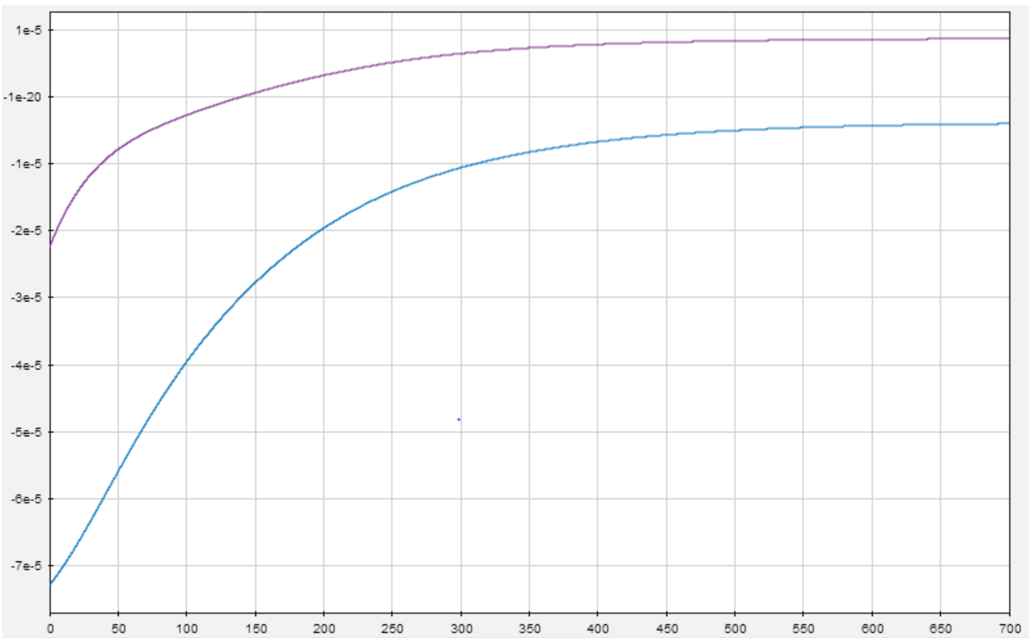

FIGURE IV.7 — The flow rates Q1 and Q2

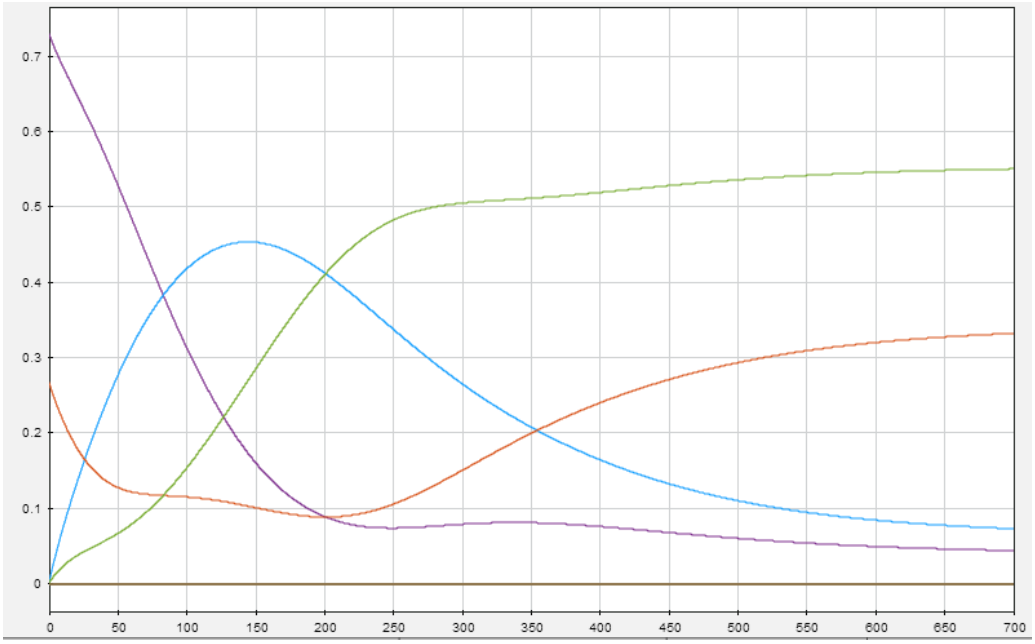

FIGURE IV.8 — The system's membership functions

IV.8 Conclusion

The main aim of this chapter was to demonstrate the interest in using Takagi-Sugeno (TS) fuzzy models for stability purposes. In order to keep it concise and easily readable, this chapter focused on the stability analysis and state feedback control for particular classes of TS models.

The first part of the chapter presents a brief overview of LMIs and their useful properties, with the objective of using this LMIs for analyze the stability and to design controllers for TS system. Then by introducing the Lyaponov's theorem as an LMI we can have a basic understanding of stability of a non linear system, we employed the Parallel Distributed Compensation (PDC) control strategy to determine the stabilizing gain for the Takagi-Sugeno (TS) fuzzy model of our hydraulic non linear system .

The PDC approach involves designing a set of parallel distributed controllers, one for each linear sub-model of the TS fuzzy system. These parallel controllers are then combined using the same membership functions as the TS model, resulting in a globally stabilizing nonlinear controller for the overall system.

By employing the LMI region methods (adding the LMI region constraints) we where able to generate the right command gains that with the right eigenvalues lead our system to stability, In other words , we can finally say that our quest in archiving the stability (stabilisation) of this hydraulic system is finally fulfilled .

GENERAL CONCLUSION AND OUTLOOK

GENERAL CONCLUSION AND OUTLOOK

This thesis focused on modeling, stability analysis, and control design for multi-variables nonlinear systems using the Takagi-Sugeno (TS) fuzzy approach. The motivation behind employing TS fuzzy models stemmed from the limitations of linear control methods when dealing with large operating ranges and uncertainties in nonlinear systems.This project afforded us the opportunity to delve into the intricate challenges posed by intricate and nonlinear systems, while equipping us with a formidable arsenal of powerful tools to confront them head-on.
We have meticulously developed an accurate TS model for our hydraulic system, paving the way for a profound understanding of its intrinsic behavior and unlocking the doors to advanced control potentialities. Our exploration of the stability and stabilization of TS models, particularly through the lens of PDC control, has illuminated the path towards maintaining the coherence and robustness of our system .
To demonstrate the effectiveness of the TS fuzzy approach, a complex hydraulic three-tank system was modeled using this technique. The resulting TS model was validated by comparing its behavior with the dynamic model derived from the underlying physical laws. Simulations conducted in MATLAB revealed an excellent match between the two models, with negligible differences (errors), demonstrating the accuracy and applicability of the TS fuzzy modeling approach for the studied system .
Subsequently, the thesis delved into the analysis of stability and control design for TS models. Linear Matrix Inequalities (LMIs) played a crucial role in this process, enabling the formulation of stability and control problems as convex optimization problems. Lyapunov's direct method was employed to establish stability and stabilization results, primarily focusing on quadratic stability for simplicity and ease of expressing problems in LMI format.
The Parallel Distributed Compensation (PDC) control strategy was applied to design a stabilizing controller for the TS fuzzy model of the hydraulic system. This approach involved designing parallel distributed controllers for each linear sub-model and combining them using the same membership functions as the TS model, resulting in a globally stabilizing nonlinear controller for the overall system.

In summery, this thesis demonstrated the effectiveness of the TS fuzzy approach for modeling, stability analysis, and control design of nonlinear systems. The proposed methods were successfully applied to a complex hydraulic system, highlighting the potential of TS fuzzy models in capturing nonlinear dynamics accurately while leveraging the well-established linear control techniques. The results pave the way for further exploration and application of TS fuzzy models in various domains involving nonlinear control challenges.This work is a testament to an enduring quest for efficiency and precision in the automation and control of dynamic systems, thereby making a significant contribution to the ever-evolving landscape of modern engineering.

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Abstract:

This thesis explores the Takagi-Sugeno (TS) fuzzy modeling approach for accurately representing and stabilizing a multivariable nonlinear hydraulic system. The TS fuzzy model employs a multi-model architecture composed of linear sub-models combined through fuzzy rules to capture the overall nonlinear behavior. An accurate TS fuzzy model of a complex three-tank hydraulic system is developed and validated against a conventional dynamic model. Stability analysis and control design techniques using linear matrix inequalities (LMIs) and Lyapunov's direct method are investigated. A parallel distributed compensation (PDC) control strategy is applied to design a stabilizing controller for the TS fuzzy model. The proposed methods demonstrate the effectiveness of the TS fuzzy approach for modeling and control of multivariable nonlinear hydraulic systems, overcoming limitations of linear techniques while providing an intuitive multi-model framework.

Keywords: Takagi-Sugeno fuzzy modeling, multi-model control , parallel distributed compensation, stabilization of nonlinear systems .

Résumé :

Cette thèse explore l'approche de modélisation floue Takagi-Sugeno (TS) pour représenter avec précision et stabiliser un système hydraulique non linéaire multivariable. Le modèle flou TS emploie une architecture multi-modèle composée de sous-modèles linéaires combinés à travers des règles floues pour capturer le comportement non linéaire global. Un modèle flou TS précis d'un système hydraulique à trois réservoirs complexe est développé et validé par rapport à un modèle dynamique conventionnel. Les techniques d'analyse de stabilité et de conception de contrôleur utilisant les inégalités matricielles linéaires (LMI) et la méthode directe de Lyapunov sont étudiées. Une stratégie de compensation parallèle distribuée (PDC) est appliquée pour concevoir un contrôleur stabilisant pour le modèle flou TS. Les méthodes proposées démontrent l'efficacité de l'approche floue TS pour la modélisation et le contrôle des systèmes hydrauliques non linéaires multivariables, surmontant les limites des techniques linéaires tout en fournissant un cadre multi-modèle intuitif.

Mots-clés : Modélisation floue Takagi-Sugeno, contrôle multi-modèle, compensation parallèle distribuée, stabilisation des systèmes non linéaires.